Ch. 2.1 Sample Space

Statisticians use the word *experiment* to describe any process that generates a set of data, which may be numerical or not. In the latter case, the data is called *categorical* or (nominal.)

Examples of experiments that generate data:
- tossing a coin results in two possible outcomes
- tossing 3 coins at a time results in many possible outcomes
- opinions of voters on a tax law yields

**Definition:** The set of all possible outcomes of a statistical experiment is called the **sample space** and is represented by the symbol $S$.

Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$ be a sample space for some experiment. Each $x_i$ is an outcome of the experiment, also called a member or element of the sample space or sometimes simply a sample point.

**Example 1:** For the first two of our previous examples, possible sample spaces are given next.
- tossing a coin results in two possible outcomes
  $S = \{H, T\}$ where H represents heads and T represents tails.
- tossing 3 coins at a time results in many possible outcomes.
  $S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

One helpful way to find all possible outcomes for “some” experiments is to use a **tree diagram**.

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This also hints at how we will count the number of elements in sample spaces that are similar to this (but not all sample spaces.)
Sets and Set Notation

Since sample spaces are collections of objects, the discussion of sets, set operations, and counting is very need.

Terms: The objects in a set are called the elements, or members, of the set. A set is said to contain its elements.

Notation: Even in statistics, we use upper case letters to denote sets, e.g. $A, B, C, S, X, \ldots$

Def: Two sets are equal if and only if they have the same elements.

Example 2: Let $A = \{1, 2, 5\}, B = \{1, 2, 3\}$, and $C = \{5, 2, 1\}$. Sets $A$ and $C$ are equal (note the order does not matter), but no other combinations in this example are equal.

Notation/Definitions:
1. $\in$ is read as "is an element of".
   Example: Let $A = \{1, 2, 5\}$. $1 \in A, 2 \in A$, but $3 \notin A$.

2. If a set is finite or has a pattern then the set can be described by listing the elements. But a more general way to describe a set is by the use of set builder notation (our text calls this sets described by a statement or a rule.)

Example 3:
   i. $R = \{x \mid x \text{ is a real number}\}$
      (The braces indicate a set and the vertical bar is read as "such that") This is read as "the set of all $x$ such that $x$ is a real number"

   ii. $O = \{x \mid x \text{ is an odd positive integer less than 10}\}$
      This is read as "the set of all $x$ such that $x$ is an odd positive integer less than 10".
3. When discussing sets in general, the **universal set**, which we will denote as $U$, is the set of all objects under consideration in a given problem. In probability theory, the universal set is usually a **sample space**.

4. The empty set, denoted $\emptyset$ is the set with no elements.

5. **A set $A$ is said to be a subset of set $B$** if and only if every element of $A$ is also an element of $B$. The notation we use is $A \subseteq B$.

Example 4: Let $A = \{a, b, c\}$, $B = \{a, b, c, d\}$, $C = \{b, c, d\}$, and $D = \{c, b, a\}$.

**Fill in the blank with the subset symbol $\subseteq$ or the not a subset of symbol $\not\subseteq$.**

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Ch. 2.2 Events

For any given experiment we may be interested in the occurrence of certain events rather than in the outcome of a specific element in the sample space or in all elements of the sample space.

For example, suppose that our experiment is to toss 3 coins and that we are interested in the event that exactly two of the coins land on heads. Then

\[ S = \{ HHH, HHT, HTH, THH, TTT, TTH, THT, HTT \} \]

and

\[ E = \{ HHT, HTH, THH \} \] is the event that we are interested in.

**Definition:** An event is a subset of a sample space.

**Example 1:** Given the sample space \( S = \{ t \mid t \geq 0 \} \), where \( t \) is the life in years of a certain electronic component, then the event \( A \) that the component fails before the end of the fifth year is the subset \( A = \{ t \mid 0 \leq t < 5 \} \).

**Note:** since an event is a subset of a sample space, the event can be empty or the entire sample space.

Many of the events that we will discuss are compound events, that is the descriptions involve the words “not”, “and” and “or” for example. This prompts the discussion of set operations the complement, intersection and the union.
Set Operations

The **complement of a set** $A$, denoted by $A'$, is defined as follows: Let $U$ be the universal set. The $A' = \{x \in U \mid x \notin A\}$, that is the set of elements that are not in the set $A$.

Visualize with **Venn Diagrams**. Let the rectangle represent the universal set, and the shaded are the elements of the set.

The **intersection of sets $A$ and $B$** is denoted $A \cap B$ and defined as follows: $A \cap B = \{x \in U \mid (x \in A) \text{ and } (x \in B)\}$.

**Example 2**: Shade in the area that corresponds to $A \cap B$.

**Example 3**: Determine $A \cap A' = \____________$.  

**Example 4**: Let $P$ be the event that a person selected at random while dining at a popular cafeteria is a taxpayer, and let $Q$ be the event that the person is over 65 years of age. Describe the following in English:

i. $P'$:

ii. $P \cap Q$:

iii. $P' \cap Q$: 
**Definition:** Two sets $A$ and $B$ are said to be disjoint (or mutually exclusive) if $A \cap B = \emptyset$.

**Example 5:** True or False: Sets $A$ and $A'$ are disjoint.

The **union of sets $A$ and $B$** is denoted $A \cup B$ and defined as follows: $A \cup B = \{ x \in U \mid (x \in A) \text{ or } (x \in B) \}$.

**Example 6:** Shade in the area that corresponds to $A \cup B$.

**Example 7:** Determine $A \cup A' =$ __________.

**Example 8:** Let $U = \{ a, b, c, d, e, f \}$, $A = \{ a, b, c \}$ and $B = \{ b, c, d, e \}$. Determine the following:

i. $A' =$ 

ii. $B' =$ 

iii. $A'' =$ 

iv. $A \cup B =$ 

v. $A \cap B =$ 

vi. $A' \cap B' =$ 

vii. $A' \cup B' =$ 

viii. $(A \cup B)' =$ 

ix. $A \cap \emptyset =$
Example 9: Shade in the area that corresponds to $A' \cap B'$.

Extra Homework: you should use Venn Diagrams similarly as above to convince yourself of the many set identities that we will need in the discussion of sample spaces and events.

See p. 29:

1. $A \cap \emptyset = \emptyset$
2. $A \cup \emptyset = A$
3. $A \cap A' = \emptyset$
4. $A \cup A' = S$
5. $S' = \emptyset$
6. $\emptyset' = S$
7. $(A')' = A$
8. $(A \cap B)' = A' \cup B'$
9. $(A \cup B)' = A' \cap B'$