Trigonometry Preliminaries for Calculus

Certain topics from Trigonometry often plague students in Calculus. A review of these topics has been prepared and may be of value to you. Please try the following problems and compare your answers to those given. Then attend a review session at which a math instructor will discuss the solutions and stay to answer student questions. The schedule and location of the review session will be posted.

Unless told otherwise, do not use a calculator or refer to notes.

1. Write the \((x,y)\) coordinates of each point \(A, B, C, D, E\) on the unit circle. Give exact values, using radicals as necessary.

2. Give the exact value of the sine, cosine, tangent, and secant of each of the following:
   a) \(\frac{5\pi}{6}\)  b) \(\frac{5\pi}{4}\)  c) \(\frac{5\pi}{3}\)  d) \(\frac{5\pi}{2}\)  e) \(5\pi\)

3. Write each of the following as a function of \(x\):
   a) \(\sin(\cos^{-1}x)\)  b) \(\tan(\sin^{-1}x)\)  c) \(\cos(\tan^{-1}x)\)

4. Verify the identity: \(\sec x = \sqrt{\tan^2 x + 1}\)

5. Find all exact solutions with \(0 \leq x < 2\pi\) for each of the following:
   a) \(\sqrt{3} - 2 \sin x = 0\)
   b) \(\sin 2x - \sin x = 0\)
   c) \(\cos^2 x - \sin^2 x = 0\)
   d) \(\sin 2x = 0\)

6. Use a scientific calculator to find all solutions with \(0 \leq x < 2\pi\) for each of the following. Give answers correct to the nearest thousandth.
   a) \(\sin^2 x = 0.4\)
   b) \(\cos 2x = -0.6\)
   c) \(\tan x = 3 \sin x\)
Answers

1. A \((1,0)\)
B \((\sqrt{3}/2, 1/2)\)
C \((\sqrt{2}/2, \sqrt{2}/2)\)
D \((1/2, \sqrt{3}/2)\)
E \((0,1)\)

2. 
\[
\begin{array}{cccccc}
\theta & \sin \theta & \cos \theta & \tan \theta & \sec \theta \\
\hline
a) & \frac{5\pi}{6} & 1/2 & -\sqrt{3}/2 & -\sqrt{3}/3 & -2\sqrt{3}/3 \\
b) & \frac{5\pi}{4} & -\sqrt{2}/2 & -\sqrt{2}/2 & 1 & -\sqrt{2} \\
c) & \frac{5\pi}{3} & -\sqrt{3}/2 & 1/2 & -\sqrt{3} & 2 \\
d) & \frac{5\pi}{2} & 1 & 0 & \text{undef.} & \text{undef.} \\
e) & 5\pi & 0 & -1 & 0 & -1
\end{array}
\]

3. a) \(\sin(\cos^{-1} x) = \sqrt{1 - x^2}\)

b) \(\tan(\sin^{-1} x) = \frac{x}{\sqrt{1 - x^2}}\)

c) \(\cos(\tan^{-1} x) = \frac{1}{\sqrt{1 + x^2}}\)

4. \[
\sqrt{\tan^2 \theta + 1} = \sqrt{\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta}} = \sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}} = \sqrt{\frac{1}{\cos^2 \theta}} = \frac{1}{\cos \theta} = \sec \theta
\]
so \(\sqrt{\tan^2 \theta + 1} = \sec \theta\)

5. a) \{ \frac{\pi}{3}, \frac{2\pi}{3} \}

b) \{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \}

c) \{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \}

d) \{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \}

6. a) \{ 0.685, 2.457, 3.826, 5.598 \}

b) \{ 1.107, 2.034, 4.249, 5.176 \}

c) \{ 0, 1.231, 3.142, 5.052 \}