1. (3 pts) **Find examples of sets** $A$, $B$ and $C$ that such that $A - C = B - C$ but $A \neq B$.

Let $A = \{1, 2, 3\} \quad \text{Then} \quad A - C = \{1, 2, 3\} = B - C$ but $A \neq B$

2. (9 pts) Let $i$ be a positive integer and let $A_i = \{i, i + 1, i + 2, \ldots, 2i\}$. Assume $n$ is greater than 1. **List the elements** of the following sets.

i. $A_3 = \{3, 4, 5, 6\}$

ii. $\bigcap_{i=2}^{4} A_i = \emptyset$

iii. $\bigcap_{i=2}^{500} A_i = \emptyset$

iv. $\bigcup_{i=1}^{4} A_i = \{1, 2, 3, 4, 5, 6, 7, 8\}$

v. $\bigcup_{i=1}^{500} A_i = \{1, 2, 3, \ldots, 500, 501, \ldots, 1000\}$

3. (8 pts) **Prove** the set identity $\overline{A \cup (B \cap C)} = C \cap (B \cap \overline{A})$ for any sets $A$, $B$ and $C$ using set identities (provide reasons at each step).

$$
\overline{A \cup (B \cap C)} = \overline{A} \cap \overline{(B \cap C)} \\
= \overline{A} \cap \overline{B} \cap \overline{C} \\
= (B \cap C) \cap \overline{A} \\
= (C \cap B) \cap \overline{A}
$$
4. (5 points) Consider the proposition that \( A \cup B \subseteq A \cup B \cup C \). For each of the following, indicate whether it is the first line of a direct proof of the proposition, indirect proof the proposition, proof by contradiction the proposition, or none of these. Note: this problem assumes you know the definition of subset.

a. Let \( x \in A \cup B \). 
\[ \text{direct} \]

b. Let \( x \in A \cup B \cup C \). 
\[ \text{NONE} \]

c. Suppose \( x \in A \cup B \) and \( x \in A \cup B \cup C \). 
\[ \text{NONE} \]

d. Suppose \( x \in A \cup B \) and \( x \notin A \cup B \cup C \). 
\[ \text{contradiction} \]

e. Suppose \( x \notin A \cup B \cup C \). 
\[ \text{indirect} \]

5. (11 pts) Let \( Z \) be the set of integers, and \( Z^+ \) be the set of positive integers. Let \( E \) be the set of even integers, and \( E^+ \) be the set of positive even integers. Let \( S \) be the set of all finite length bit strings.

i. Let \( f: Z \to Z^+ \), where \( f(x) = x^2 \). Explain why this assignment of integers to the positive integers is NOT a one-to-one function.
\[ f(1) = 1^2 = (-1)^2 = f(-1) \]
But \( 1 \neq -1 \)

ii. Let \( f: S \to E^+ \cup \{ 1 \} \), where if \( x \) is a bit string of length \( n \), then \( f(x) = 2^n \). Use this function to answer parts a – d.
   a. Evaluate \( f(001) = 2^3 \)
   b. Explain why this function is NOT onto \( E^+ \cup \{ 1 \} \).
\[ 0 \text{ is even but not equal to } 2^n \text{ for any positive } n. \]
   c. Explain why this function is NOT one-to-one.
\[ f(001) = 2^3 = f(100) \]
But \( 001 \) is not \( 100 \)
   d. TRUE or FALSE: This function is invertible.
\[ b/c \text{ is if not } 1-1, \text{ not invertible} \]
6. (8 pts) Let $f$ be the real value function $f(x) = \lfloor x/3 \rfloor$.

   i. Evaluate $f(1) = \begin{cases} 0 \end{cases}$

   ii. Evaluate $f(-1.4) = \begin{cases} -1 \end{cases}$\[\lfloor -1.4/3 \rfloor = -1\]

   iii. The image of 7 is $\begin{cases} 2 \end{cases}$\[\lfloor 7/3 \rfloor = 2\]

   iv. All pre-images of 2 are $\begin{cases} x \mid 6 \leq x < 9 \end{cases}$

   v. Let $S$ be the set of real numbers strictly between 0 and 6 (i.e. in the interval $(0,6)$). Find the image of the set $S$, i.e. find $f(S)$.

      \[
      \begin{array}{ll}
      0 < x < 3 & f(x) = 0 \\
      3 \leq x < 6 & f(x) = 1
      \end{array}
      \]

   vi. Let $S = \{1, 2\}$. Find the pre-image of the set $S$, i.e. find $f^{-1}(S)$.

      Since $3 \leq x < 6$ \[f(x) = 1\]

5. (6 pts) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ both be onto. Fill in the blanks for the proof that $g \circ f: A \rightarrow C$ is onto.

   \[\begin{array}{ll}
   \text{Proof Let } y \in C. \text{ Since } g \text{ is onto, there exists an } x \text{ in } B \text{ such that } g(x) = y. \text{ And since } f \text{ is onto, there exists a } z \text{ in } A \text{ such that } f(z) = x. \text{ Now by definition of composition and by substitution we see that } (g \circ f)(z) = g(f(z)) = y. \text{ This completes our proof that } g \circ f \text{ is onto, since we have proven that for any } y \in C \text{ there exists a } z \in A \text{ such that } (g \circ f)(z) = y. \quad \square
   \end{array}\]

7. (7 pts) Let $f: R \rightarrow (-1,0]$ such that $f(x) = \lfloor x \rfloor - x$.

   i. Evaluate $f(2) = \begin{cases} 2 \end{cases}$\[2 - 2 = 0\]

   ii. Evaluate $f(2.5) = \begin{cases} 2.5 \end{cases}$\[2.5 - 2.5 = 0.5\]

   iii. Evaluate $f(-2.5) = \begin{cases} -2.5 \end{cases}$\[-2.5 - (-2.5) = -0.5\]

   iv. Evaluate $f(1.75) = \begin{cases} 1.75 \end{cases}$\[1.75 - 1.75 = -0.75\]

   v. Evaluate $f(-1.75) = \begin{cases} -1.75 \end{cases}$\[-1.75 - (-1.75) = 0.25\]

   vi. Find an $x$ such that $f(x) = -0.367$

      \[x = 1.367\]

   vii. TRUE or FALSE: $f$ is onto.
9. (10 pts) Suppose \( f : \mathbb{R} \rightarrow \mathbb{R} \) that has the rule \( f(n) = 3n - 7 \).
   
   (a) **Prove** that this function is one-to-one.

   **Proof:** Let \( x, y \in \mathbb{R} \). Assume \( f(x) = f(y) \).
   
   \[
   3x - 7 = 3y - 7
   \]
   
   Substitute \( 3x = 3y \) and solve for \( x \):
   
   \[
   3x = 3y \\
   x = y
   \]
   
   Thus \( x = y \). Therefore, \( f \) is 1-1. \( \Box \)

   (b) **Prove** that this function is onto \( \mathbb{R} \).

   **Proof:** Let \( y \in \mathbb{R} \). Since \( \frac{y + 7}{3} \in \mathbb{R} \) and
   
   \[
   f\left(\frac{y + 7}{3}\right) = 3\left(\frac{y + 7}{3}\right) - 7 = y,
   \]
   
   \( f \) is onto \( \mathbb{R} \). \( \Box \)

10. (6 pts) **Sequences**

   i. **List the first 4 terms** of the sequence \( \left\{ \frac{2}{i + 3} \right\}_{i=0} \)

   \[
   \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}
   \]

   ii. **List the first 4 terms** of the arithmetic sequence determined by \( a = 2 \) (initial value) and \( d = 5 \) (the common difference).

   \( a, 7, 12, 17 \)

   iii. **List the first 4 terms** of the geometric sequence determined by \( a = 2 \) (initial value) and \( r = 5 \) (the common ratio).

   \( 2, 10, 50, 250 \)
11. (6 pts) Sequence formulas
   a) For the list of integers provided find a formula that generates a sequence that begins with the list of integers.
   \[ \frac{1}{3}, \frac{2}{3^2}, \frac{3}{3^3}, \frac{4}{3^4}, \frac{5}{3^5}, \ldots \]
   \[ a_n = \frac{n}{3^n} \]

   b) Provide a formula that generates a sequence that begins with the list.
   \[ 8, 98, 998, 9998, 99998, \ldots \]
   \[ a_n = 10^n - 2 \]

   c) Provide a formula that generates a sequence that begins with the list.
   \[ 8, 13, 18, 23, 28, \ldots \]
   \[ a_n = 8 + 5(n - 1) \]

12. (6 pts) Summations
   i. Evaluate the sum \[ \sum_{k=0}^{5} 3k = 3\cdot0 + 3\cdot1 + 3\cdot2 + 3\cdot3 + 3\cdot4 + 3\cdot5 = 45 \]

   ii. Evaluate the sum \[ \sum_{k=0}^{5} 3 = 3 + 3 + 3 + 3 + 3 + 3 = 3\cdot6 = 18 \]

   iii. Evaluate the product \[ \prod_{k=0}^{5} 3 = 3\cdot3\cdot3\cdot3\cdot3\cdot3 = 3^6 \]

10. (2 points) Express \( \sum_{k=0}^{98} (k + 3) \) as a single equivalent summation whose lower limit is 2. That is find the upper limit and the argument so that \( \sum_{k=0}^{98} (k + 3) = \sum_{j=2}^{100} (j+1) \).

    \[ j = k + 2 \]
    \[ j + 1 = k + 3 \]
13. (15 pts) Determine the indicated graph parameters for the indicated graphs.

Let $G$ be the graph and use it to fill in the values in the first two rows of the table.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Number of vertices</th>
<th>Degree sequence</th>
<th>maximum degree</th>
<th>minimum degree</th>
<th>Number of edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>6</td>
<td>2, 2, 3, 3, 3, 4</td>
<td>4</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Complement of $G$</td>
<td>6</td>
<td>3, 3, 3, 2, 2, 1</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>$K_6$, the complete graph on 6 vertices</td>
<td>6</td>
<td>5, 5, 5, 5, 5, 5</td>
<td>5</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

14. (3 pts) Recall that the graph called the $n$-Cube is the graph whose vertex set is the set of $2^n$ bitstrings of length $n$, with two vertices adjacent if and only if their respective bitstring representations differ in exactly one position. Below is the drawing of the 3-cube. Label the vertices of the drawing so that it is clear that this is the 3-cube.