Math 1301 - Test 3 - Spring 2008

Name: Solutions **Student ID:** _____

1. (12 Pts) In parts 1a and 1b, fill in each blank with the correct answer.

1a. For a Quadratic function, the formula can be written in the form $f(x) = ax^2 + bx + c$ where a is not zero.

1b. The graph of the function $f(x) = -x^2 - 4x - 5$ is a parabola that has lead coefficent = -1, y-intercept = -5, x-intercepts = N/A and vertex at $(-2, -1)$.

x-intercepts

$0 = -x^2 - 4x - 5$

$0 = x^2 + 4x + 5$

$x = \frac{-4 \pm \sqrt{16 - 4(1)5}}{2 \cdot 1} = \frac{-4 \pm \sqrt{-4}}{2}$

No real solution

Vertex

$x = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$

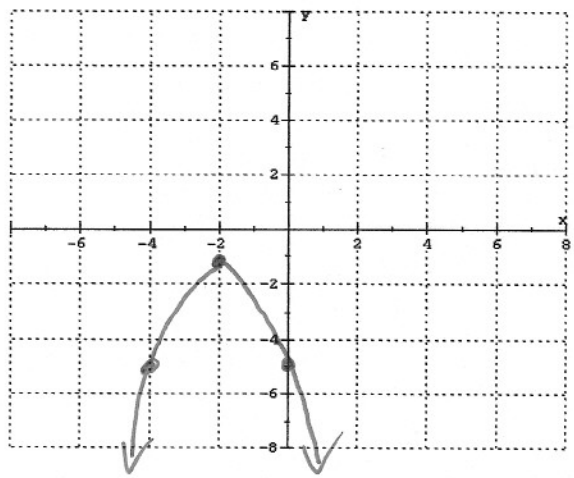
$y = f(-2) = -(-2)^2 - 4(-2) - 5 = -4 + 8 - 5 = -1$

x	y
-4	$-16+16-5$

1c. By hand, sketch the graph of the function

$f(x) = -x^2 - 4x - 5$

on the set of axes given here.
Plot sufficient points to get a good graph.



2. (8 Pts) Write the quadratic function $f(x) = x^2 + 6x - 3$ in vertex-form by completing the square.

$y = x^2 + 6x - 3$

$y + 3 = x^2 + 6x$

$y + 3 + 9 = x^2 + 6x + 9$

$y + 12 = (x+3)^2$

$y = (x+3)^2 - 12$

$\left(\frac{6}{2}\right)^2 = 9$

5. (16 Pts) Find the exact solutions of each quadratic equation symbolically (by hand) and indicate the number of real solutions and the number of complex solutions. Show your work.

5a. $3(2x+7)(5x-1)=0$ The solution(s) are $-7/2,\ 1/5$

There are 2 real solutions and 0 complex solutions

5b. $\sqrt{(x+3)^2}=\sqrt{5}$ The solution(s) are $\pm\sqrt{5}-3$

There are 2 real solutions and 0 complex solutions

$x+3=\pm\sqrt{5}$

$x=\pm\sqrt{5}-3$

5c. $4x^2=x+1$ The solution(s) are $(1\pm\sqrt{17})/8$

There are 2 real solutions and 0 complex solutions

$4x^2-x-1=0$

$$\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot4(-1)}}{2\cdot4}=\frac{1\pm\sqrt{17}}{8}$$

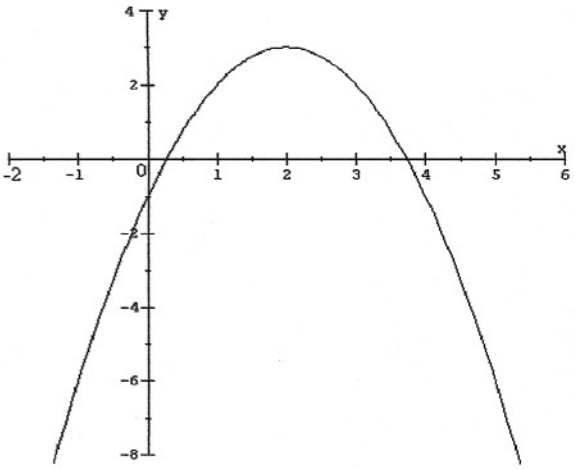
5d. $4x^2=x-1$ The solution(s) are $\frac{1\pm\sqrt{15}\cdot i}{8}$

There are 0 real solutions and 2 complex solutions

$4x^2-x+1=0$

$$\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot4\cdot1}}{2\cdot4}=\frac{1\pm\sqrt{-15}}{8}=\frac{1\pm\sqrt{15}\,i}{8}$$

3. (12 Pts) The graph of a quadratic function $f(x)$ is given here. Use this graph to answer the following questions about the function f. Assume that the vertex and the y-intercept have integer coordinates.



3a. The sign of the leading coefficient is Negative (positive or negative).

3b. The vertex has coordinates (2 , 3)

3c. So far we have that $f(x) = a(x - 2)^2 + 3$

3d. The $y-$ intercept is (0 , -1)

3e. Substitute the $y-$ intercept into the function in part c in order to find the lead coefficient a.

$x=0 \quad f(x)=-1$

$-1 = a(0-2)^2+3$

$-1 = 4a+3$

$-4 = 4a$

$a = -1$

3f. Now write the function that fits the graph $f(x) = -1\cdot(x-2)^2+3$

$-(x^2-4x+4)+3$

$-x^2+4x-4+3$

$-x^2+4x-1$

4. (10 Pts) A baseball is hit straight up. The height of the baseball is a function of the elapsed time after it is hit. Suppose the height $s(t)$ in feet after t seconds is given by

$$s(t) = -16t^2 + 112t + 3$$

4a. Write a sentence to interpret the practical meaning of the statement $s(7) = 0$.

3 After 7 seconds the ball is on the ground

4b. Write a sentence to interpret the practical meaning of the statement $s(0) = 3$.

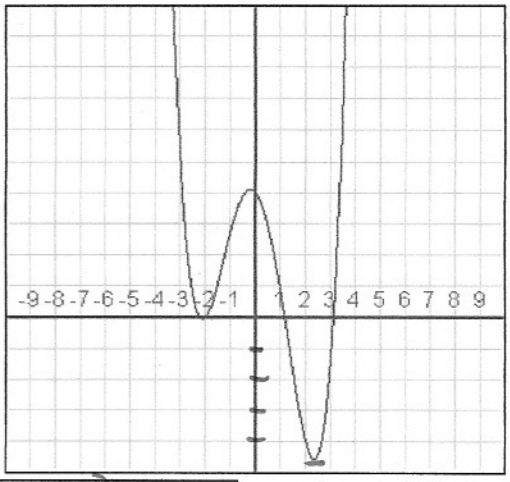
3 Before the ball is hit the ball is 3 ft off the ground

4 **4c.** Find the maximum height of the baseball and the time needed to reach this height.

← max at Vertex $\quad \frac{-b}{2a} = \frac{-112}{2(-16)} = 3.5$ secs

$s(3.5) = -16(3.5)^2 + 112(3.5) + 3 = 199$ feet

6. (13 pts) Study the graph of the *polynomial* function f given here.



6a. *Exactly* how many x-intercepts does f have? 3

6b. *Exactly* how many turning points does f have? 3

6c. What is the smallest degree that this function *could* have? 4

6d. *True* or *False*: The lead coefficent is positive.

6e. What is the absolute minimum of the function f? $(2.5, -4.5)$

6f. What is the absolute maximum of the function f? N/A

6g. What is/are the local minimum of the function f? $(-2, 0)$; $(2.5, -4.5)$

6h. What is/are the local maximum of the function f? $(-.5, 4)$

6i. What is the range of f? Write the answer in interval notation. $[-4.5, \infty)$

6j. Find the interval(s) where the function f is increasing. Write the answer in interval notation.
$(-2, -0.5), (2.5, \infty)$

7. (12 pts)

a. Write the expresion in terms of i. $\sqrt{-49} =$ $7i$

$\sqrt{-49} = \sqrt{-1 \cdot 49} = \sqrt{-1} \cdot \sqrt{49} = i \cdot 7 = 7i$

b. Perform the operation and write the answer in the $a + bi$ form. $10i$

$\sqrt{-25} + 5i = 5i + 5i$

c. Perform the operation and write the answer in the $a + bi$ form. $2i - 11$

$(-2 - 3i) - (9 - 5i) = -2 - 3i - 9 + 5i$

d. Perform the operation and write the answer in the $a + bi$ form. $9 - 17i$

$(2 - i)(7 - 5i)$

$14 - 10i - 7i + 5(-1)$

8. (5 pts) Write the number in the $a + bi$ form. $\frac{4}{2-5i}$

$8/29 + 20/29\, i$

$$\frac{4}{(2-5i)} \cdot \frac{(2+5i)}{(2+5i)} = \frac{8+20i}{4-25i^2} = \frac{8+20i}{4-25(-1)} = \frac{8+20i}{29}$$

9. (12 pts) **To answer parts a-e** consider the rational function $f(x) = \frac{2}{x-4}$

a. Find the y-intercept.

$y = -1/2$

$$y = f(0) = \frac{2}{0-4} = -\frac{1}{2}$$

b. Find the x-intercept.

N/A

$$\frac{2}{x-4} = 0 \quad \text{No solution}$$

c. Find the domain of f.

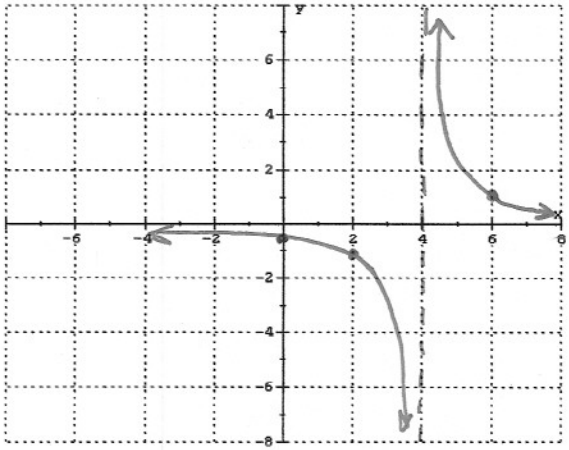
$(-\infty, 4) \cup (4, \infty)$

$$x - 4 \neq 0$$
$$x \neq 4$$

d. True or False: The point $(2, -1)$ is on the graph of f.

$$f(2) = \frac{2}{2-4} = \frac{2}{-2} = -1$$

e. Sketch the graph and use a dashed line to indicate any vertical asymptotes..



x	y
6	1