

Ch. 4.3 Nondecimal Bases

H10: 2-32 evens

In our everyday life we use the decimal notation, base ten, to represent any real number.

Example: $2054 = 2(10^3) + 0(10^2) + 5(10^1) + 4(10^0)$

Computers use base two (binary)

Example: $(1010)_2 = 1(2^3) + 0(2^2) + 1(2^1) + 0(2^0)$

For any positive integer $b \geq 2$, we have a base b system for representing numbers. In defining a base b system we allow for only b symbols (in base 10, we use only $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and in base 2 we use only $\{0, 1\}$.) We define place values right to left for example as follows:

$$\begin{array}{cccccc} \overline{10^4} & \overline{10^3} & \overline{10^2} & \overline{10^1} & \overline{10^0} & \text{for base 10} \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & \text{for base 2} \end{array}$$

Theorem 4.8 Let b be a positive integer greater than or equal to 2. Then every positive integer a can be expressed uniquely in the form

$$a = a_k b^k + a_{k-1} b^{k-1} + a_{k-2} b^{k-2} + \dots + a_1 b + a_0 b^0,$$

where a_0, a_1, \dots, a_k are nonnegative integers less than b , $a_k \neq 0$, and $k \geq 0$.

Example 1: Express in base 10

i) $(1010)_2$

ii) $(7142)_8$

Suppose that we are given a base 10 number and need to convert to some other base.

Example 2: Express 35 in base 2.

We know that $2^5 = 32$, and $35 = 32 + 2 + 1$, so

$$35 = 1(2^5) + 0(2^4) + 0(2^3) + 0(2^2) + 1(2^1) + 1(2^0) = (100011)_2$$

There is an algorithm, in which we repeated divide by the base value and keep track of remainders.

$35 \div 2$ yields quotient 17 remainder 1

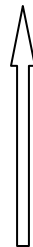
$17 \div 2$ yields quotient 8 remainder 1

$8 \div 2$ yields quotient 4 remainder 0

$4 \div 2$ yields quotient 2 remainder 0

$2 \div 2$ yields quotient 1 remainder 0

$1 \div 2$ yields quotient 0 remainder 1



Example 3: Express 676 in base 8.

Base 16: Note that if we use a base larger than 10 then we run into a dilemma with the symbols. For instance we cannot use $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ for base 16 since we would cause ambiguity as to what $(1015)_{16}$ would mean. Thus for base 16 we use $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

Example 4: Express $(A52D)_{16}$ in base 10.

Base b arithmetic.

Recall base 10 arithmetic

$$\begin{array}{r} 145 \\ + 78 \\ \hline \end{array}$$

$$\begin{array}{r} (101)_2 \\ + (1)_2 \\ \hline \end{array}$$

Simple Addition Table Base 2

+	0	1
0		
1		

Example 5: Add the binary integers 10110_{two} and 1011_{two} .

Multiplication base b . (Note that we shift just as in base 10)

11_{two} times 101_{two} .

$$\begin{array}{r}
 101 \\
 \times 11 \\
 \hline
 101 \quad \leftarrow \text{multiply } 101 \text{ by } 1 \\
 101 \quad \leftarrow \text{multiply } 101 \text{ by } 1 \\
 \hline
 1111
 \end{array}$$

Binary Subtraction

Let us look at some simple examples through conversion

$$10_{\text{two}} - 1_{\text{two}} = 2 - 1 = 1 = 1_{\text{two}}$$

$$11_{\text{two}} - 1_{\text{two}} = 5 - 1 = 4 = 10_{\text{two}}$$

$$100_{\text{two}} - 1_{\text{two}} = 8 - 1 = 7 = 111_{\text{two}}$$

Definition: The **one's complement** x' of a binary number x is obtained by replacing each 0 in x with a 1 and replacing each 1 in x with a 0.

Examples: Practice the definitions; assume we are using base 2

x	x' (one's complement)	$x' + 1$ (two's complement)
10	01	10
11	00	01
100	011	100
11001	00110	00111

Example 6: Subtract 1011_{two} from 100001_{two} .

Note: 100001 is called the minuend and 1011 is called the subtrahend.

Step 1: pad subtrahend with zeros on the left if necessary so that it has the same number of digits as the minuend. Then find the one's complement of the (possibly padded) subtrahend.

1011 gets padded with 2 zeros, which yields 001011 . Then find the one's complement which is 110100 .

Step 2: add one to the result of step 1 (i.e find the two's complement of the (padded) subtrahend).

$$110100 + 1 = 110101$$

Step 3: add the two's complement (from step 2) to the minuend.

Step 4: delete the leading 1. The resulting number is the answer.