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Graffiti.pc Red Burton Style - A Student's Perspective

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The purpose of this paper is a discussion of my observations while conducting undergraduate research, formally my senior project, on the concept of the alpha-core number of a graph using the program Graffiti.pc. Graffiti.pc was written by Dr. Ermelinda DeLaVina at the University of Houston-Downtown. It takes as input a database of user-defined graphs and user-specified concepts and returns graph theoretic conjectures on a user-chosen concept. This research was structured by a version of the Red Burton rules. Dr. Siemion Fajtlowicz of the University of Houston created the Red Burton rules as a method of resolving conjectures created by the programs Graffiti (and consequently Graffiti.pc). Dr. Fajtlowicz discusses and explains these rules in this paper [**A**]. The version of the Red Burton rules which were used in my research are outlined as follows:

1. The first conjecture to appear on the list will be resolved. (Note that in Graffiti.pc, if the conjectures remain unsorted by touch number then it is usually the case that the first is the most simply stated conjecture).
2. If the resolved conjecture is false then find the minimum number of vertices in a counterexample, and next the minimum number of edges of a counterexample with the minimum number of vertices. In this case the counterexample is added to the database.
3. If the resolved conjecture is true then characterize the case of equality and determine if one can verify in polynomial time that a graph has the characterization described. In the case such a characterization is accomplished, graphs from the class are forbidden from the database; further, any counterexamples for subsequent conjectures could not be in this class of graphs. Otherwise, the next conjecture on the list is resolved.

Prior to beginning the research, I spent approximately six months conducting similar research on the sum of the independence number of a graph and the clique number of a graph. This initial research was my first exposure to graph theory, and was conducted in the spirit of the Moore method of teaching. I was given the basic definition of the concepts with which I would be working, but no further information. Consequently, as this research progressed I rediscovered many theorems which

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are presented in an introductory graph theory class. Also, I began to develop a set of skills which aided in resolving future conjectures, such as looking for a family of counterexamples, or a smallest counterexample, and characterizing the graphs for which equality holds in the case of a true inequality. Due to the unique process of discovery in this research, I was able to make some interesting observations about this learning experience in comparison to a traditional lecture-based class.

First, the structure of the Red Burton rules allowed me to develop an understanding of the larger picture. When the project began, there were many occasions when Dr. DeLaVina explained the larger picture. However, my understanding did not fully develop until I was able to see the pieces fitting together through the rules. The goal of the research project was to obtain, if possible, a 'formula' for the alpha-core number of a graph in terms of invariants of the degree sequence of a graph. With each true statement, once the graphs for which equality held were characterized, a piece of the larger picture was put in place. This direct evolution allowed me to much more fully comprehend the overall goal.

For a simple connected graph G, let $\alpha_c(G)$ denote the alpha-core number, which we defined to be the cardinality of the intersection of all maximum independent sets of the graph G. Let $\overline{G}$ denote the complement of graph G, K_m the complete graph on m vertices, D_m the graph on m vertices with no edges, $E(G)$ the edge set of the graph G and Length($\overline{G}$) the square root of the sum of the squares of degrees. What follows is the main result of my senior project, which is a partial result of the goal of the project.

$$\alpha_c(G) = \begin{cases} \left|E(G)\right| & \text{if } G \simeq join(K_1, D_m) \text{ for } m \geq 2; \\ 2\left|E(\overline{G})\right| & \text{if } G \simeq K_m \text{ or } join(K_m, D_2) \text{ for } m \geq 2; \\ \lfloor \text{Length}(\overline{G}) \rfloor & \text{if } G \simeq join(K_m, D_3) \text{ for } m \geq 2 \text{ or} \\ & join(K_m, K_{3,2}) \text{ for } m \geq 0. \\ ? & \end{cases}$$

In a traditional class, there are usually standard topics which are covered. Unfortunately, often the instructor may not have the time to make certain that students fit these topics into a larger picture. Consequently, the larger picture may be lost, in which case the student may be left with what appears to be individual pieces of information. The critical difference that I found between this research and a standard classroom was the factor of my personal involvement. I was not given each piece of a puzzle and then told where to place it. I got the opportunity to carve a few pieces and fit them into place myself.

Secondly, the process of finding these pieces had an element which could have been seen as a drawback to the Red Burton rules, but instead became another important learning experience not often found in a traditional classroom setting. On a few occasions, there were statements which were found to be true for a larger class of graphs than the one stated in the hypothesis. That is, it became important to critically examine the full statement of a conjecture to determine whether the resulting theorem was the strongest possible statement that could be proven. In a standard class setting, this opportunity would most likely never occurs. Even if there is an uncertainty as to the truth of the given statement, the actual wording of the statement is not questioned. For example, one conjecture I examined was that the alpha-core number of a graph is not more than one plus the maximum degree over all vertices in the complement of the graph. When this conjecture appeared, there were several restrictions already placed on the class of graphs. However, once I determined that this conjecture is, in fact, true, and began to try to classify the class of graphs for which equality held, Dr. DeLaVina pointed out that the statement was true for all graphs and that no restrictions were required. As in this case, once a student begins research, it is likely that examining the statement and the conditions imposed on the particular case will be an important part of the process. This early opportunity to gain experience was a valuable lesson.

Finally, it is always important to understand the tools one uses. The program Graffiti.pc was used for this research, and while I have not studied the heuristics of Graffiti and Graffiti.pc, directly interacting with the program Graffiti.pc has allowed me to begin to see rationale for the appearance of the conjectures. Two aspects of this research directly helped in my understanding of the program as well as seeing how the same thought process could be used in future work. The first was the Red Burton rules themselves. Since only one conjecture is resolved on each list, there are, perforce, others which are left untouched. Watching these conjectures either continue to the next list or disappear entirely gave some insight into what bounds continued to be viable options. The second aspect ties closely to the first and reinforces its effect. In the course of this research, I had the opportunity to examine three very similar series of conjectures regarding the alpha-core number of a graph, each of which used slightly different versions of the Red Burton rules. Of course, since the initial assumptions and databases where identical, very similar and sometimes identical conjectures were found in all three trials. Examining the three trials side by side gave even further insight for me into the heuristics of the program as well as suggested a final result may be found, regardless of the route which is taken.

The observations made in this paper have been just a brief discussion of some of the larger benefits obtained while conducting the discussed research. A more detailed discussion of the program Graffiti.pc can be found in Dr. DeLaVina's paper [**B**], and for a full description of the research conducted and the results obtained see my paper [**C**]. While this type of research may not replace traditional classroom learning, the opportunity to think a little differently and to practice skills not usually presented to undergraduate students was an invaluable experience.

References

[A] S. Fajtlowicz, *Toward Fully Automated Fragments of Graph Theory*, Graph Theory Notes of New York, XLII:2 (2002), 18-25.

[B] E. DeLaVina, *Graffiti.pc.*, Graph Theory Notes of New York XLII:3 (2002), 26-30.

[C] B. Chervenka, *Graph Theory Graffiti/Graffiti.pc Style, Senior Project Report.* (2001), University of Houston - Downtown.

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