

Logic

Def. A **Proposition** is a statement that is either true or false.

Examples: Which of the following are propositions?

Statement	Proposition (yes or no)	If yes, then determine if it is true or false.
UHD has a brick exterior		
$1 + 3 = 0$		
$2 + 3$		
$x + 7 = 18$		
$x + 7 = 18$ for $x = 9$		
$x + 7 = 18$ for every real number x		

Notation: We use lower case letters to denote propositions.

Def. **Compound propositions** are new propositions formed from existing propositions using *logical operators*.

Note: We will discuss the following *logical operators* (also called connectives)

- Negation operator, $\neg$ (Other books use $\sim$)
- Conjunction operator, $\wedge$
- Disjunction operator, $\vee$
- Exclusive or, $\oplus$
- Implication, $\rightarrow$
- Biconditional, $\leftrightarrow$

Def. Let p be a proposition. The statement "it is not the case that p " is a proposition formed from p and the negation operator, called **the negation of p** , which we denote by $\neg p$. This proposition is read as "not p ". The truth-value of $\neg p$ is true when p is false and false when p is true.

Truth Table

p	$\neg p$
T	F
F	T

Examples:

1. Let p be the statement " $7 + 1 = 6$ ". Write the statement $\neg p$ as an English statement. What is the truth-value of $\neg p$?
2. Let p be the statement "*It is not Friday*". Write the statement $\neg p$ as an English statement. What is the truth-value of $\neg p$?

Def. Let p and q be propositions. The statement " p and q " is a proposition formed from p , q and the conjunction operator, called **the conjunction of p and q** , which we denote by $p \wedge q$. The truth-value of $p \wedge q$ is true when both p and q are true, otherwise $p \wedge q$ is false.

Truth Table

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

Def. Let p and q be propositions. The statement " p or q " is a proposition formed from p , q and the disjunction operator, called **the disjunction of p and q** , which we denote by $p \vee q$. The truth-value of $p \vee q$ is false when both p and q are false, otherwise $p \vee q$ is true.

Truth Table

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

Examples: Let p be the statement "*Houston is the capital of Texas*" and let q be the statement "*Houston is a city in Texas*". Determine the following:

- a. The truth-value for $p \wedge q$.
- b. The truth-value for $p \vee q$.
- c. The truth value for $\neg p \wedge q$
- d. The truth value for $\neg(p \wedge q)$
- e. The truth value for $\neg p \vee q$

Def. Let p and q be propositions. The statement " p or q exclusively" is a proposition formed from p , q and the exclusive or operator, called the exclusive or of p and q , which we denote by $p \oplus q$. The truth-value of $p \oplus q$ is true when exactly one of p or q is true, otherwise $p \oplus q$ is false.

Truth Table

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

Comparison: A familiar example of the use of an "exclusive or" is a restaurant menu. "Price of the entree includes soup or salad". Note that one or the other is included in the price *but not both*. A familiar example of the use of inclusive or is the prerequisite for this course. "Students who have taken calculus or computer science, can take this course." Note that if a student has taken both courses, then the student may still take this course.

Def. An implication (or conditional statement) is statement of the form "if p then q ", denoted $p \rightarrow q$. Such propositions are also read as " p implies q ". The "if part" is called the hypothesis (or premise), and the "then part" is called the conclusion (or conclusion). The implication if p then q is false only when the hypothesis is true and the conclusion is false.

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Terminology: An implication $p \rightarrow q$ may be worded in several different ways:

- If p , then q
- If p , q
- q if p
- p only if q
- p is sufficient for q
- q is necessary for p

Examples: Determine the truth-values for the following:

- If 2 is an even integer, then today is Friday.
- If $3 + 3 = 7$, $1 + 1 = 2$.
- $1 + 1 = 2$ if $3 + 3 = 7$.

Def. Given the implication $p \Rightarrow q$:

$q \Rightarrow p$ is the **converse** of the implication $p \Rightarrow q$.

$\neg p \Rightarrow \neg q$ is the **inverse** of implication $p \Rightarrow q$.

$\neg q \Rightarrow \neg p$ is the **contrapositive** of implication $p \Rightarrow q$.

Example: Write the converse, the inverse, and the contrapositive of the given implication:

If $x \in A$, then $x \in A \cap B$	Proposition	True or False?
	Converse	True or False?
	Inverse	True or False?
	Contrapositive	True or False?

Def. The **biconditional proposition** denoted by $p \leftrightarrow q$ is the conjunction of $p \rightarrow q$ and $q \rightarrow p$. The proposition $p \leftrightarrow q$ is read as p if and only if q . The biconditional proposition is true when both p and q have the same truth-values and is false otherwise.

p	q	$p \leftrightarrow q$
T	T	
T	F	
F	T	
F	F	

Examples: Determine the truth-values for the following:

- The number 2 is an odd integer if and only if today is Friday. _____
- If $3 + 3 = 7$, $1 + 1 = 2$ and conversely if $1 + 1 = 2$, $3 + 3 = 7$. _____

Exercise. Let p and q be the propositions

p : It is below freezing.

q : It is snowing

Write the following propositions using the symbols p and q and any appropriate logical connectives.

Use the following (a) –(d) to fill in the blank for i) and ii)

(a) $p \wedge q$ (b) $p \vee q$ (c) $p \rightarrow q$ (d) none of these

i) It is below freezing and snowing. _____

ii) It is below freezing if it is snowing _____

Use the following (a) –(d) to fill in the blank for iii) - vi)

(a) $p \oplus q$ (b) $p \vee q$ (c) $p \rightarrow q$ (d) none of these

iii) It is not below freezing and it is not snowing. _____

iv) It is either snowing or below freezing (or both). _____

v) It is either snowing or below freezing (but not both). _____

vi) That it is below freezing is necessary and sufficient for it to be snowing. _____

Exercise #24 Construct a truth table for each of the following compound propositions.

c) $(p \wedge q) \vee r$

e) $(p \vee q) \wedge \neg r$