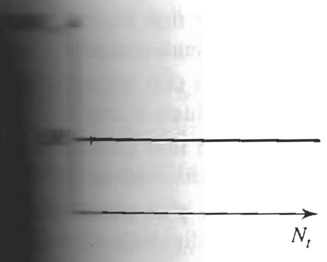
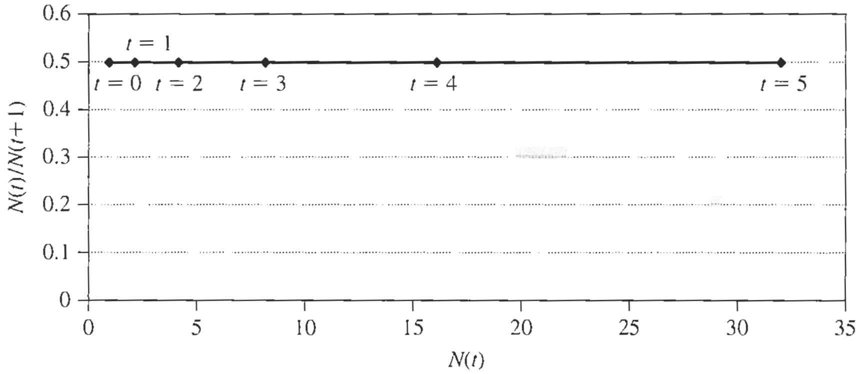


Figure 2.8
The graph of the parent to offspring ratio $\frac{N_t}{N_{t+1}}$ as a function of N_t when $N_t > 0$



▲ **Figure 2.9**
The graph of the parent to offspring ratio $\frac{N_t}{N_{t+1}}$ as a function of N_t when $N_t = 1$ and $R = 2$

We also see from Figure 2.7 that unless we label the points according to the corresponding t-value, we would not be able to tell at what time a point (N_t, N_{t+1}) was realized. We say that time is *implicit* in this graph. Compare this to Figure 2.3, where we graphed N_t as a function of t for the same values of R and N_0; in Figure 2.3, time is *explicit*.

The hallmark of exponential growth is that the ratio of successive population sizes, N_t/N_{t+1}, is constant. When $N_t > 0$ (and hence $N_{t+1} > 0$), it follows from $N_{t+1} = RN_t$ that

$$\frac{N_t}{N_{t+1}} = \frac{1}{R}$$

If the population consists of annual plants, we can interpret the ratio N_t/N_{t+1} as the parent-offspring ratio. If this ratio is constant, parents produce the same number of offspring regardless of the current population density. Such growth is called **density independent**.

When $R > 1$, then $1/R$, the parent-offspring ratio, is less than 1, implying that the number of offspring exceeds the number of parents. Density-independent growth with $R > 1$ results in an ever-increasing population size. This eventually becomes biologically unrealistic since any population will sooner or later experience food or habitat limitations that will limit its growth. We will discuss models that include such limitations in Section 2.3.

The density independence in exponential growth is reflected in a graph of N_t/N_{t+1} as a function of N_t, which is a horizontal line at level $1/R$ (Figure 2.8).

As before, only a selected number of points are realized on the graph of N_t/N_{t+1} as a function of N_t and time is implicit in the graph. See Figure 2.9 when $R = 2$ and $N_0 = 1$.

Problems

In Problems 1–12, produce a table for $n = 0, 1, 2, \ldots, 5$, and graph the function.

1. $f(n) = \frac{1}{n+2}$
2. $f(n) = \frac{1}{1+n^2}$
3. $f(n) = \frac{1}{(1+n)^2}$
4. $f(n) = \frac{1}{\sqrt{n+1}}$
5. $f(n) = n^2 - 1$
6. $f(n) = n^3 + 1$

7. $f(n) = (n+1)^2$

8. $f(n) = \sqrt{n+4}$

9. $f(n) = e^{\sqrt{n}}$

10. $f(n) = 3e^{-0.1n}$

11. $f(n) = \left(\frac{1}{3}\right)^n$

12. $f(n) = 2^{0.2n}$

13. A strain of bacteria reproduces asexually every hour. That is, every hour, each bacterial cell splits into two cells. If initially there is one bacterium, find the number of bacterial cells after one hour, two hours, three hours, four hours, and five hours.

14. A strain of bacteria reproduces asexually every 30 minutes. That is, every 30 minutes, each bacterial cell splits into two cells. If initially there is one bacterium, find the number of bacterial cells after one hour, two hours, three hours, four hours, and five hours.

15. A strain of bacteria reproduces asexually every 23 minutes. That is, every 23 minutes, each bacterial cell splits into two cells. If initially there is one bacterium, how long will it take until there are 128 bacteria?

16. A strain of bacteria reproduces asexually every 42 minutes. That is, every 42 minutes, each bacterial cell splits into two cells. If initially there is one bacterium, how long will it take until there are 512 bacteria?

17. A strain of bacteria reproduces asexually every 10 minutes. That is, every 10 minutes, each bacterial cell splits into two cells. If initially there are three bacteria, how long will it take until there are 96 bacteria?

18. A strain of bacteria reproduces asexually every 50 minutes. That is, every 50 minutes, each bacterial cell splits into two cells. If initially there are 10 bacteria, how long will it take until there are 640 bacteria?

19. Find the exponential growth equation for a population that doubles in size every unit of time and that has forty individuals at time 0.

20. Find the exponential growth equation for a population that doubles in size every unit of time and that has 53 individuals at time 0.

21. Find the exponential growth equation for a population that triples in size every unit of time and that has 20 individuals at time 0.

22. Find the exponential growth equation for a population that triples in size every unit of time and that has 72 individuals at time 0.

23. Find the exponential growth equation for a population that quadruples in size every unit of time and that has five individuals at time 0.

24. Find the exponential growth equation for a population that quadruples in size every unit of time and that has 17 individuals at time 0.

25. Find the recursion for a population that doubles in size every unit of time and that has 20 individuals at time 0.

26. Find the recursion for a population that doubles in size every unit of time and that has 37 individuals at time 0.

27. Find the recursion for a population that triples in size every unit of time and that has 10 individuals at time 0.

28. Find the recursion for a population that triples in size every unit of time and that has 84 individuals at time 0.

29. Find the recursion for a population that quadruples in size every unit of time and that has 30 individuals at time 0.

30. Find the recursion for a population that quadruples in size every unit of time and that has 62 individuals at time 0.

In Problems 31–34, graph the functions $f(x) = a^x$, $x \in [0, \infty)$ *and* $N_t = R^t$, $t \in \mathbf{N}$ *together in one coordinate system for the indicated values of* a *and* R.

31. $a = R = 2$

32. $a = R = 3$

33. $a = R = 1/2$

34. $a = R = 1/3$

In Problems 35–46, find the population sizes for $t = 0, 1, 2, \ldots, 5$ *for each recursion.*

35. $N_{t+1} = 2N_t$ with $N_0 = 3$

36. $N_{t+1} = 2N_t$ with $N_0 = 5$

37. $N_{t+1} = 3N_t$ with $N_0 = 2$

38. $N_{t+1} = 3N_t$ with $N_0 = 7$

39. $N_{t+1} = 5N_t$ with $N_0 = 1$

40. $N_{t+1} = 7N_t$ with $N_0 = 4$

41. $N_{t+1} = \frac{1}{2}N_t$ with $N_0 = 1024$

42. $N_{t+1} = \frac{1}{2}N_t$ with $N_0 = 4096$

43. $N_{t+1} = \frac{1}{3}N_t$ with $N_0 = 729$

44. $N_{t+1} = \frac{1}{3}N_t$ with $N_0 = 3645$

45. $N_{t+1} = \frac{1}{5}N_t$ with $N_0 = 31250$

46. $N_{t+1} = \frac{1}{4}N_t$ with $N_0 = 8192$

In Problems 47–58, write N_t *as a function of* t *for each recursion.*

47. $N_{t+1} = 2N_t$ with $N_0 = 15$

48. $N_{t+1} = 2N_t$ with $N_0 = 7$

49. $N_{t+1} = 3N_t$ with $N_0 = 12$

50. $N_{t+1} = 3N_t$ with $N_0 = 3$

51. $N_{t+1} = 4N_t$ with $N_0 = 24$

52. $N_{t+1} = 5N_t$ with $N_0 = 17$

53. $N_{t+1} = \frac{1}{2}N_t$ with $N_0 = 5000$

54. $N_{t+1} = \frac{1}{2}N_t$ with $N_0 = 2300$

55. $N_{t+1} = \frac{1}{3}N_t$ with $N_0 = 8000$

56. $N_{t+1} = \frac{1}{3}N_t$ with $N_0 = 3500$

57. $N_{t+1} = \frac{1}{5}N_t$ with $N_0 = 1200$

58. $N_{t+1} = \frac{1}{7}N_t$ with $N_0 = 6400$

In Problems 59–66, graph the line $N_{t+1} = RN_t$ *in the* N_t*-*N_{t+1} *plane for the indicated value of* R *and locate the points* (N_t, N_{t+1}), $t = 0, 1,$ *and* 2 *for the given value of* N_0.

59. $R = 2$, $N_0 = 2$

60. $R = 2$, $N_0 = 3$

61. $R = 3$, $N_0 = 1$