If $\mu$ is the mean and $\sigma$ is the standard deviation, the $z$-score of a value $x$ is

$$z = \frac{x - \mu}{\sigma}.$$  

**Joint distributions**

- Let $X$ and $Y$ be jointly discrete with joint probability mass function $p(x, y)$. Then
  - marginal probability mass functions for $X$ and $Y$ are defined as
    $$p_X(x) = \sum_y p(x, y) \quad \text{and} \quad p_Y(y) = \sum_x p(x, y)$$
  - conditional probability density functions are given by
    $$p_{X|Y}(x|y) = \frac{p(x, y)}{p_Y(y)} \quad \text{and} \quad p_{Y|X}(y|x) = \frac{p(x, y)}{p_X(x)}$$
  - conditional expectations are given by
    $$E(X|Y = y) = \sum_x x p_{X|Y}(x|y) \quad \text{and} \quad E(Y|X = x) = \sum_y y p_{Y|X}(y|x)$$
  - if $h(X, Y)$ is any function, then
    $$\mu_{h(X,Y)} = \sum_x \sum_y h(x, y)p(x, y)$$

- Let $X$ and $Y$ be jointly continuous with joint probability density function $f(x, y)$. Then
  - if $a, b, c$ and $d$ are arbitrary real numbers, then
    $$P(a < X < b \text{ and } c < Y < d) = \int_a^b \int_c^d f(x, y)dydx$$
  - marginal probability density functions for $X$ and $Y$ are defined as
    $$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy \quad \text{and} \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$$
  - conditional probability density functions are given by
    $$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)} \quad \text{and} \quad f_{Y|X}(y|x) = \frac{f(x, y)}{f_X(x)}$$
  - conditional expectations are given by
    $$E(X|Y = y) = \int_{-\infty}^{\infty} x f_{X|Y}(x|y)dx \quad \text{and} \quad E(Y|X = x) = \int_{-\infty}^{\infty} y f_{Y|X}(y|x)dy$$
  - if $h(X, Y)$ is any function, then
    $$\mu_{h(X,Y)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y)f(x, y)dxdy$$
If $X$ and $Y$ are jointly distributed, then
- covariance of $X$ and $Y$ is given by

$$Cov(X, Y) = \mu_{XY} - \mu_X \mu_Y$$

- correlation of $X$ and $Y$ is given by

$$\rho_{X,Y} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

If $X$ is a discrete random variable with probability mass function $p(x)$, then

$$\mu_X = \sum_x x p(x) \quad \text{and} \quad \sigma^2_X = \sum_x (x - \mu)^2 p(x).$$

If $X$ is a continuous random variable with probability density function $f(x)$, then

$$\mu_X = \int_{-\infty}^{\infty} x f(x) dx \quad \text{and} \quad \sigma^2_X = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx.$$