On the Enumeration of a Class of Non-Graceful Graphs (⋆)

Adam Drake and Timothy A. Redl

Abstract

A simple graph $G$ is a graceful graph if there exists a graceful labeling of the vertices of $G$. If we cannot gracefully label the vertices of $G$, then $G$ is a non-graceful graph. A result by Rosa provides a sufficient condition for a graph to be non-graceful: “If a graph $G$ is simple, even, and has $e$ edges, with $e \equiv 1$ or 2 (mod 4), then $G$ is not graceful.” This condition implies an infinite subclass of non-graceful graphs, which we define to be $\mathcal{R}$. By the degree-sum formula for graphs, the sum of the degrees of a graph $G$ is equal to $2e$. We systematically enumerate graphs in $\mathcal{R}$ by first generating all even partitions of $2e$ (where $e \equiv 1$ or 2 (mod 4)) using Maple. We then use the Havel-Hakimi procedure to determine which of these generated “number sequences” are graphic sequences. These graphic sequences determine all of the simple, even, graphs in $\mathcal{R}$ (both connected and disconnected) with $e \equiv 1$ or 2 (mod 4) edges.

Key Words: graceful labeling, graceful graph, non-graceful graph, graphic sequence