Functions

What is a function?

Example 1

If you buy some oranges, the total price you pay is exactly determined by the amount of oranges that you buy. In other words, *the price is a function of the amount of oranges purchased*. To be more specific, suppose the oranges cost $0.68 per lb.

- How much does it cost for 2 lb of oranges?

$$0.68 \times 2 = 1.36 \text{ dollars}$$

- How much does it cost for 3 1/2 lb of oranges?

$$0.68 \times 3\frac{1}{2} = 2.38 \text{ dollars}$$

- How much does it cost (p dollars) for A lb of oranges?

$$0.68 \times A = p \text{ dollars}$$

Example 1

- Is total price a function of the number of oranges purchased?
- No, because smaller oranges cost less than larger oranges. A person buying ten small oranges will pay less than a person buying ten large oranges, even though both buy the same number of oranges. Therefore, the total price is not exactly determined by the number of oranges purchased.

Example 2

- The following table provides guidelines for a working student can use when deciding the number of credit hours to take in a term, given the amount of time they must work each week.

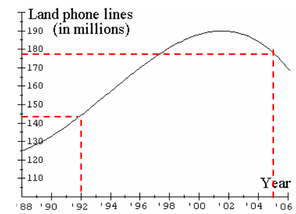
Number of hours worked	0	10	20	30	40
Maximum credit hours	16	13	10	7	4

- This table shows *the maximum recommended number of credit hours a typical student should enroll for as a function of the number of hours worked each week.*
  - How many credit hours should Joe Nobody enroll for if he plans on working 20 hours each week?

  The answer is: Joe Nobody should enroll not more than 10 credit hours.

Example 3

- As cellular phones have become more popular with consumers, the number of land phone lines in the U.S. has started to decrease. Analysts have devised the following graph that shows the trend in the number of land phone lines in the U.S.
- This graph relates the number of land phone lines (in millions) to time (measured in years). According to the graph, *the number of land phone lines is a function of the year*.
- How many land phone lines were in 1992?
  The answer is: There were 144 land phone lines in 1992.
- How many land phone lines will be in 2005?
  The answer is: There will be 176 land phone lines in 2005.



Definition of Function

- A *function* is any rule, method or system that can be used to predict or determine the value of an unknown quantity based on the value of a known quantity
- A *function* can be defined as an *input-output process* in which each input number results in one and only one output value.
- More simply, a function matches each input with exactly one output.
- The 3 (or 4) fundamental ways to write/represent a function are:
  - A table of input-output pairs
  - A graph of a set of points
  - A formula/symbol rule
  - A verbal description

Examples

Each of the following sentences describes quantities that are related to one another. Which of these sentences describe mathematical functions? State TRUE or FALSE and briefly explain your answer.

- The number of hours of daylight in New York City is a function of the day of the year.
- The day of the year is a function of the number of hours of daylight in New York City.
- The number of hairs on an adult male's head is a function of his age.
- The number of hairs on an average adult male's head is a function of his age.
- The area of a square is a function of the length of the sides of the square.
- The length of the sides of a square is a function of the area of the square.
- The area of a rectangle is a function of the width of the rectangle.
- The width of a rectangle is a function of the area of the rectangle.
- The July electricity bill for a home in Los Angeles is a function of the size of the home.
- The average July electricity bill for a home in Los Angeles is a function of the size of the home.

Example

- Consider the function relating the area enclosed by a circle to the radius of the circle. *The area of a circle is a function of the radius because we can use the radius to compute the area.* The function rule used to determine the area based on the radius has been known since ancient times: Square the radius and then multiply by π.
- It is impossible to completely represent this area function using a table because there are too many values for the radius which must be included in the table. Even between the values 1 and 2, there are infinitely many choices for the radius of a circle.
- A graph or a symbol rule is a much better way to represent this function.

Function Notation

Consider the function

$f(x) = x^2 + 5x$ [Think: $f(\ ) = (\ )^2 + 5 \cdot (\ )$]

The "output that matches the input 1" is **expressed** symbolically as $f(1)$, read "f of 1."

Evaluate $f(1)$ means "use the rule to compute the matching output." Hence,

Problem: **Evaluate** $f(1)$.

Answer: $f(1) = (1)^2 + 5 \cdot (1) = 1 + 5 = 6$

Write your answer in function notation.

– Evaluate $f(-1)$

Problems on Page 155

Evaluate the function at the indicated values.

18. $f(x) = x^3 - 4x^2$; $f(0)$, $f(1)$, $f(-1)$, $f\left(\frac{3}{2}\right)$, $f\left(\frac{x}{2}\right)$, $f\left(x^2\right)$

Find $f(a)$, $f(a+h)$, and $\frac{f(a+h)-f(a)}{h}$ when $h \neq 0$.

32. $f(x) = \frac{1}{x+1}$

Domain of a Function

The *domain* of a function is the *set of all input numbers for the function that yield mathematically matching output values.*

Domain of a Function

- For most functions represented by symbol rules, there are two types of numbers that must be excluded from the domain:
  - First Type. Input numbers that result in an attempt to divide by 0 as the matching output is being evaluated from the symbol rule.

$$f(x) = \frac{1}{x}$$

Problem 42 p. 156

Find domain of the function *f*

$$f(x) = \frac{1}{3x - 6}$$

Domain of *f* =

Example

Given the following function *f*

$$f(x) = \frac{1}{(x-2)(x+1)}$$

Domain of *f* =

Domain of a Function

- Second Type. Input numbers that result in extracting an even root of a negative number as the matching output is being evaluated from the symbol rule. (By "even" root we mean square root, fourth root, etc.)

$$f(x) = \sqrt{x}, \quad f(x) = \sqrt[4]{x}, \quad f(x) = \sqrt[6]{x}$$

Example

$$h(t) = \sqrt{5 - 2t}$$

Domain of h =

Problem 63 on page156

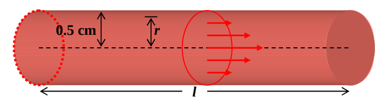
As blood moves through a vein or an artery, its velocity v is greatest along the central axis and decreases as the distance r from the central axis increases (see the figure). The formula that gives v as a function of r is called the **law of laminar flow**. For an artery with a radius 0.5 cm, we have

$$v(r) = 18{,}500(0.25 - r^2) \quad 0 \le r \le 0.5$$

a) Find $v(0.1)$ and $v(0.4)$.

b) What do your answer to part a) tell you about the flow of blood in this artery?

c) Make a table of values of $v(r)$ for r = 0, 0.1, 0.2, 0.3, 0.4, 0.5



Problems on page156

Find domain of the function f.

48. $f(x) = \sqrt{7 - 3x}$

54. $f(x) = \sqrt{x^2 - 2x - 8}$

Practice Problems on page155

13,16,17,29,30,35,37-53 odd, 61,62,66.

Graphs of Functions

Types of Functions

- Constant functions √
- Polynomial functions √
  - Linear functions
  - Quadratic functions
- Variation (proportion) functions √
- Functions defined by one symbol rule or piecewise functions √
- Exponential functions √
- Logarithmic functions √
- Rational functions √
- Radical functions

Graph of Functions

The graph of a function f is a set of all order pairs or points $(x, f(x))$ or (x, y) where $y = f(x)$.

Notes:

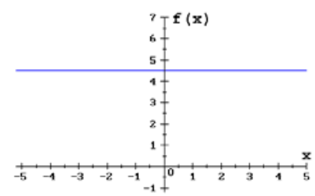
- Properties of functions can be revealed from their graphs.
- If the point (x_1, y_1) is on the graph of f, then $y_1 = f(x_1)$.
- Given the graph of f and a value of x as x_2, then $f(x_2)$ can be evaluated by analyzing the given graph of f.

Constant Functions

$$f(x) = c$$

- Example:

$$f(x) = 4.5$$

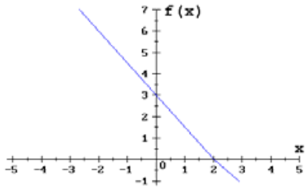


Linear Functions

$f(x) = mx + b$

- Example:

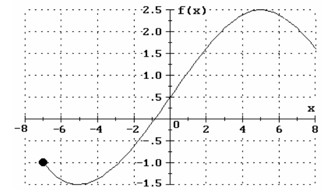
$f(x) = -\frac{3}{2}x + 3$



Polynomial Functions

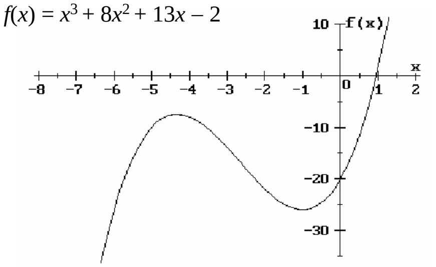
Use the function represented by the graph on the right to evaluate each of the given function outputs. (Note: The output may not be defined.)

a) $f(0)$
b) $f(-3)$
c) $f(7)$
d) $f(-8)$
e) $f(8)$



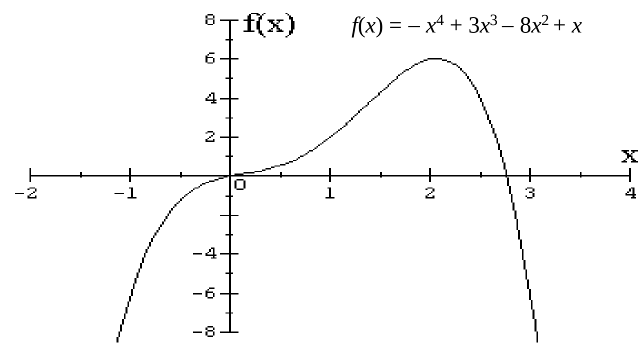
Polynomial Functions

$f(x) = x^3 + 8x^2 + 13x - 2$



Polynomial Functions

$f(x) = -x^4 + 3x^3 - 8x^2 + x$



Piecewise Functions

Dry Gulch Arizona

To encourage conservation, the city of Dry Gulch, Arizona, has approved a new rate structure for water usage for its residential customers. For the first 8,000 gallons used per month, the rate is \$4.25 per 1,000 gallons. When the customer uses 8,000 gallons up to 12,000 gallons, he or she pays a penalty of \$10 in addition to \$5.25 per 1,000 gallons used. For any amount over 12,000 gallons, the customer pays a penalty of \$25 in addition to \$7.25 per 1,000 gallons used. Let $P(g)$ be the amount paid by a residential customer for consuming g thousand gallons of water in a month.

$$p(g) = \begin{cases} 4.25 \cdot g, & \text{if } 0 \le g \le 8 \\ 10 + 5.25 \cdot g, & \text{if } 8 < g \le 12 \\ 25 + 7.25 \cdot g, & \text{if } g > 12 \end{cases}$$

Example

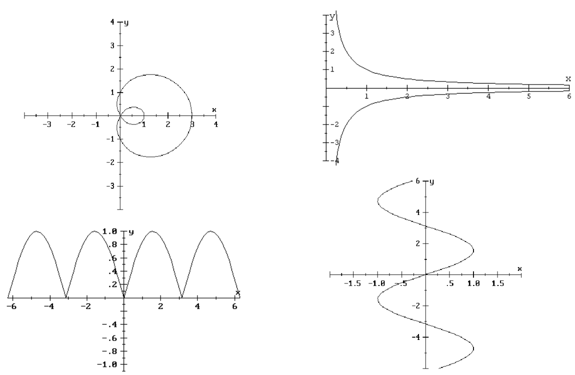
Let

$$P(x) = \begin{cases} -x^2 - 4x, & \text{if } x \le -2 \\ 4, & \text{if } -2 < x \le 2 \\ -x^2 + 4x, & \text{if } 2 < x < 5 \end{cases}$$

a) Write the domain of the function using interval notation.

b) Evaluate $P(-1)$, $P(3)$, $P(-2)$.

Determine Graphs is Function



Practice Problems on page165

2-10

Applied Functions: Variation

Direct Variation

A ***direct variation function*** has a symbol rule that can be written in the form

$$y = kx$$

where x is the input, y is the matching output, and k is a positive real number, called ***constant of proportionality***

Example

Consider the price p (in dollars) for purchasing A pounds of oranges. The price p is directly proportional to number of pounds A purchased. If the price of the oranges is $0.68 per pound,. What is the symbol rule for the function relating the price p to the number of pounds A purchased?

Example

At a constant rate of speed, the distance d traveled by an object is directly proportional to the elapsed amount of time t the object travels. The constant of proportionality k is the rate of speed of the object. For example, the distance that the International Space Station travels when making one orbit of the Earth is about 25,733 miles. The time it takes to make an orbit is about one-and-a-half hr. What is the symbol rule for the function relating the distance d traveled by the space station to the elapsed amount of time t the space station travels?

Inverse Variation Functions

A ***inverse variation function*** has a symbol rule that can be written in the form

$$y = \frac{k}{x}$$

where x is the input, y is the matching output, and k is a positive real number, called ***constant of proportionality***

Example

For a given distance, the amount of time t required to cover the distance is inversely proportional to the rate of speed r. For example, suppose a car travels from Houston to Dallas, Texas. What is the symbol rule for the function relating the time t needed for the car to cover this distance at the rate of speed r, if the car can cover this distance in 4 hr at 60 mi/hr?

Examples

For the following functions, state whether the function is a direct proportion, inverse proportion, or neither. For the proportion functions, state the value of the constant of proportionality .

1. $d = 60t$
2. $M = 0.001/n$
3. $F = 9/5\ C + 32$
4. $y = x/2$

Joint Variation Functions

A ***joint variation function*** has a symbol rule that can be written in the form

$$z = kxy \text{ or } z = k\frac{x}{y}$$

depends on whether z jointly varies directly or z varies directly as x and inversely as y, where x and y are the input, z is the matching output, and k is a positive real number, called ***constant of proportionality***

Problems on Page 172

19. C is jointly proportional to l, w, and h. If $l = w = h = 2$, then $C = 128$.

34. The rate r at which a disease spreads in a population of size P is jointly proportional to the number x of infected people and the number $P - x$ who are not infected. An infection erupts in a small town with population $P = 5000$.

a) Write an equation that express r as a function of x.
b) Compare the rate of spread of this function when 10 people are infected to the rate of spread when 1000 people are infected. Which rate is larger? By what factor?
c) Calculate the rate of spread when the entire population is infected. Why does this answer make intuitive sense?

Practice Problems on Page 172

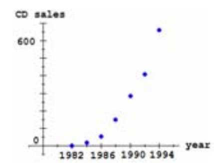
1,3,5,7,11,13,17,18,20,23,24,26,27,28,
29,31,32

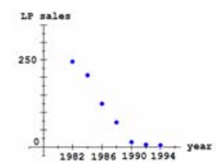
Average Rate of Change

Example

The following is a table displaying annual sales of CDs and LPs in millions from 1982 to 1994.

Year	1982	1984	1986	1988	1990	1992	1994
CD sales (millions)	0	5.8	53	150	287	408	662
LP sales (millions)	244	205	125	72	12	2.3	1.9

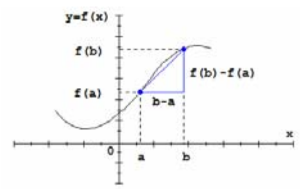




Average Rate of Change

Average rate of change of $y = f(x)$ between $x = a$ and $x = b$ is

$$\text{Average rate of change over } [a,b] = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$



Problems on Page 182

Determine the average rate of change of the function between the given values.

5. $f(x) = x^3 - 4x^2; \quad x = 0, x = 10$

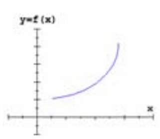
10. $f(x) = \dfrac{2}{x+1}; \quad x = 0, x = h$

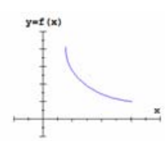
14. $f(x) = -4x + 2; \quad x = a, x = a + h$

Increasing and Decreasing Functions

$f(x)$ is *increasing* on an interval $[x_1, x_2]$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$. (That is f increases from x_1 to x_2.)

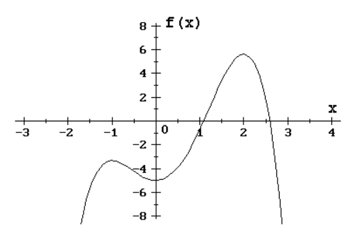
$f(x)$ is *decreasing* on an interval $[x_1, x_2]$ if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$. (That is f decreases from x_1 to x_2.)





Example

Find intervals on which f is increasing and decreasing.



Problem 21 on Page 183

Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
CD players sold	512	520	413	410	468	510	590	607	732	612	584

The table shows the number of CD players sold in a small electronics store in the year 1989-1999.

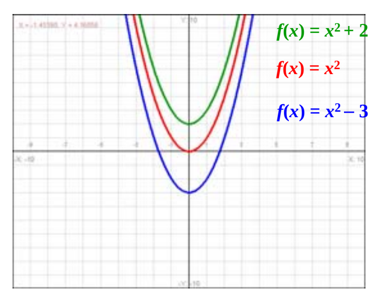
a) What was the average rate of change of sales between 1989 and 1999?
b) What was the average rate of change of sales between 1989 and 1990?
c) What was the average rate of change of sales between 1990 and 1992?
d) Between which two successive years did CD player sales increases most quickly? Decreases most quickly?

Practice Problems Page 182

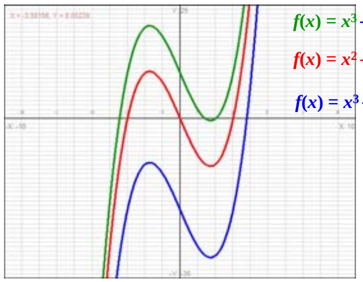
3, 4, 13, 17-19, 23-26

Transformations of Functions

Vertical Shifting



Vertical Shifting

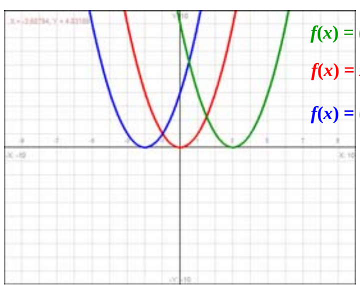


Vertical Shifting

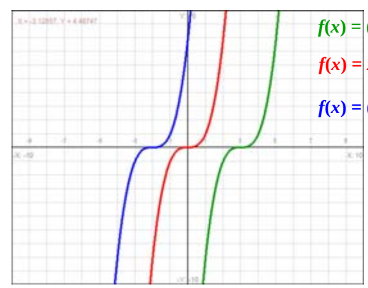
If $y = f(x)$ is a function and c is a positive constant then the graph of

- $y = f(x) + c$ is the graph of $y = f(x)$ shifted up vertically c units
- $y = f(x) - c$ is the graph of $y = f(x)$ shifted down vertically c units

Horizontal Shifting



Horizontal Shifting

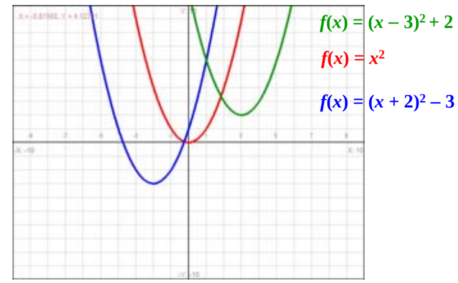


Horizontal Shifting

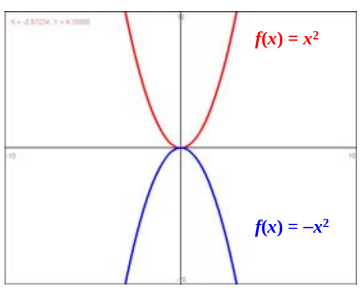
If $y = f(x)$ is a function and c is a positive constant then the graph of

- $y = f(x + c)$ is the graph of $y = f(x)$ shifted left horizontally c units
- $y = f(x - c)$ is the graph of $y = f(x)$ shifted right horizontally c units

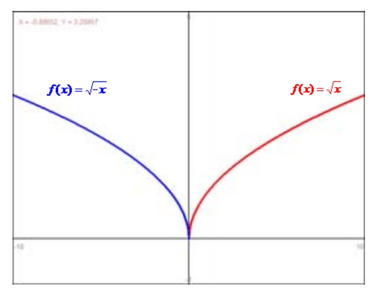
Horizontal and Vertical Shifting



Reflecting Graphs (x-axis)



Reflecting Graphs (y-axis)

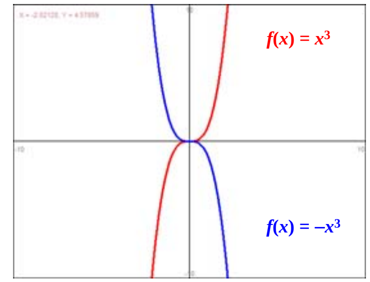


Reflecting Graphs

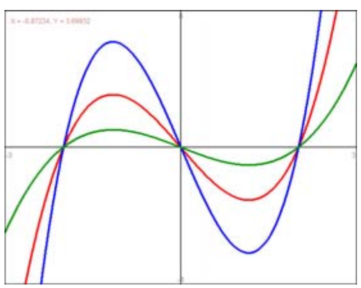
If $y = f(x)$ is a function then the graph of

- $y = -f(x)$ is the reflection the graph of $y = f(x)$ in the x-*axis*
- $y = f(-x)$ is the reflection the graph of $y = f(x)$ in the y-*axis*

Reflecting Graphs (x and y -axis)



Vertical Stretching and Shrinking

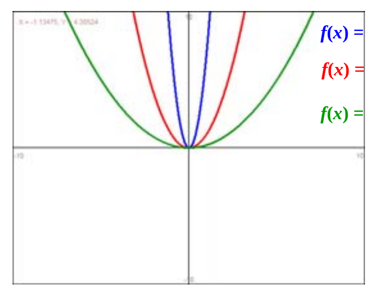


$f(x) = 2(x^3 - 4x)$

$f(x) = x^3 - 4x$

$f(x) = (1/3)\ (x^3 - 4x)$

Vertical Stretching and Shrinking



$f(x) = 7x^2$

$f(x) = x^2$

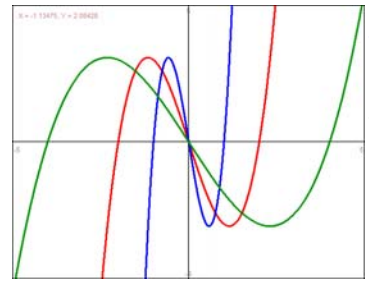
$f(x) = (1/5)\ x^2$

Vertical Stretching and Shrinking

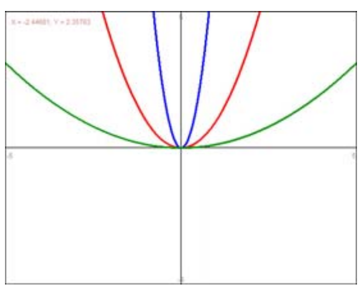
If $y = f(x)$ is a function then the graph of

- $y = a\,f(x)$ is the graph of $y = f(x)$ stretch vertically by a factor of a for $a > 1$.
- $y = a\,f(x)$ is the graph of $y = f(x)$ shrink vertically by a factor of a for $0 < a < 1$.

Horizontal Stretching and Shrinking



Horizontal Stretching and Shrinking

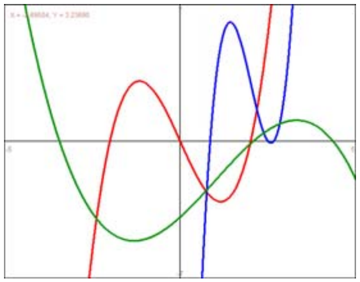


Horizontal Stretching and Shrinking

If $y = f(x)$ is a function then the graph of

- $y = f(ax)$ is the graph of $y = f(x)$ shrink horizontally by a factor of a for $a > 1$.
- $y = f(ax)$ is the graph of $y = f(x)$ stretch horizontally by a factor of a for $0 < a < 1$.

Shifting, Stretching and Reflecting

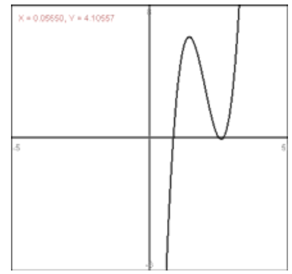


$f(x) = 8(x-2)^3 - 8(x - 2) + 3$

$f(x) = x^3 - 4x$

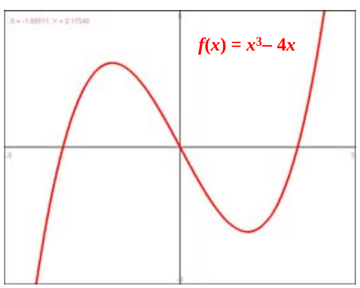
$f(x) = (1/8)(-x+1)^3 - 2(-x+1) - 2$

Shifting, Stretching and Reflecting

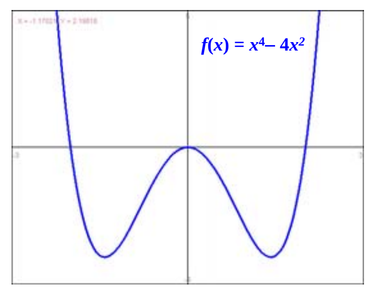


$f(x) = (2(x - 2))^3 - 4(2(x - 2)) + 3$

Even and Odd Functions



Even and Odd Functions



Even and Odd Functions

- f is *even* if $f(x) = f(-x)$ for all x in the domain of f. Then the graph is symmetric with respect to y-axis.
- f is *odd* if $f(x) = -f(x)$ for all x in the domain of f. Then the graph is symmetric with respect to the origin.

Practice Problems on Page 194

1,2,3,4,6,8,9,11,12,15,19,21,23,41,44,45,46,59,60

Extreme Values of Functions

Extreme Values of Quadratic Functions

The general form of *quadratic function* is

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real number and $a \neq 0$.

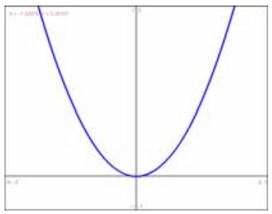
- The graph of quadratic function is *parabola.*
- The y - intercept of any quadratic function is always at $y = c$.

A simple Quadratic Function

A simple *quadratic function* is in the form of

$$f(x) = x^2$$

where $a = 1$, b, and c are zero and the graph is



From a simple Quadratic Function

Sketch the graph of the following quadratic functions

$$f(x) = x^2 + 3$$
$$f(x) = (x^2 - 2) + 3$$
$$f(x) = 4(x^2 - 2) + 3$$
$$f(x) = 4(x^2 - 2) - 3$$
$$f(x) = 4(x^2 + 2) - 3$$
$$f(x) = \frac{1}{2}(x^2 + 2) - 3$$

Graph of Quadratic Functions

Sketch the graph of the following the quadratic function $f(x) = 2x^2 - 12x + 23$

Standard form of Quadratic Functions

The *standard* form of a quadratic function is

$$f(x) = a(x-h)^2 + k$$

where h and k are real numbers and the vertex is at (h, k).

If $a > 0$, then the parabola is *concave up* and the the *minimum value* of the function f is k occurs at $x = h$, i.e., $f(h) = k$.

If $a < 0$, then the parabola is *concave down* and the the *maximum value* of the function f is k occurs at $x = h$, i.e., $f(h) = k$.

Problems on Page 204

Sketch the graph of the given the quadratic functions and state the coordinates of its vertex and its intercepts

12. $f(x) = 2x^2 + x - 6$
13. $f(x) = -4x^2 - 16x + 3$
21. $g(x) = 3x^2 - 12x + 13$

Problem

Find the vertex for the parabola of $f(x) = ax^2 + bx + c$

Minimum or Maximum Value of a Quadratic Function

The *minimum* or *maximum* value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a}$$

If $a > 0$, then the *minimum value* is $f\left(-\frac{b}{2a}\right)$

If $a < 0$, then the *maximum value* is $f\left(-\frac{b}{2a}\right)$

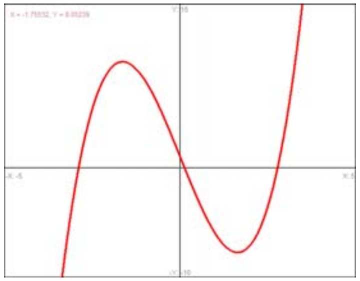
Problems on Page 204

Find the maximum or minimum value of the functions

27. $f(t) = 100 - 49t - 7t^2$
31. $f(x) = \frac{1}{2}x^2 + 2x - 6$

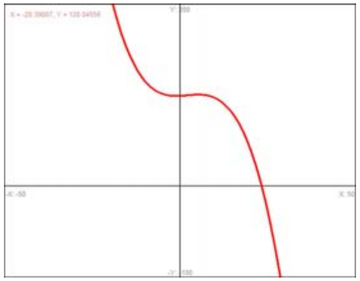
Using Graphing Device to Find Extreme Values

Example 6 p. 203 $f(x) = x^3 - 8x + 1$



Using Graphing Device to Find Extreme Values

Example 7 p. 203 $I(t) = -0.0113t^3 + 0.0681t^2 + 0.198t + 99.1$



Problems on Page 205

Find the maximum or minimum value of the functions

48. $f(x) = 3 + x + x^2 - x^3$

49. $f(x) = x^4 - 2x^3 - 11x^2$

54. $V(x) = \dfrac{1}{x^2 + x + 1}$

Practice Problems on Page 204

1,2,5,6,9,10,25,26,33,34,35,37,38,41,42.

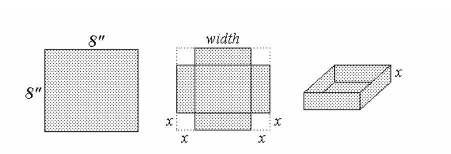
Modeling with Functions

Guidelines for Modeling with Functions

1. Gather the given information
2. Express the model in words
3. Choose the variable
4. Set up the model
5. Use the model

Example 1: Modeling the Volume of a Box

Tuan, the weekend gardener, harvested a bumper crop of jalapenõs this year. Tuan's wife, Ermelinda, desperately looking for new ways to use the jalapenõs, invented jalapenõ-flavored popcorn. Tuan finds this popcorn so delicious that he thinks he can market it commercially. He has decided to start up a garage business to sell Ermelinda's Three-Alarm Jalapenõ Popcorn. The popcorn kernels will be sold in 2-lb boxes. After experimenting, Tuan discovers that 2 lb of the popcorn kernels occupies a volume of 25 cubic inches. Tuan and Ermelinda decide to make the boxes themselves from surplus 8-in. × 8-in. pieces of cardboard. They will cut the same size square from each corner of a piece of cardboard and then fold up the sides to form an open box. They will then wrap each popcorn-filled box with clear plastic and apply a label.



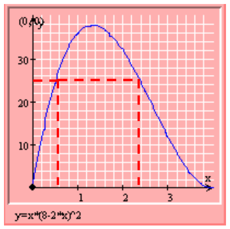
Example 1: Modeling the Volume of a Box

- Volume = *height* × *width* × *length*
- $V(x) = x \times 8 - 2x\ 8 - 2x$
  $= x(8 - 2x)^2$
- $V(1) = 1(8 - 2\times 1)^2$
  $= 36 \text{ in}^3$
- Interpret the value $V(1)$ using a complete sentence.
- If the length of a side of the square cut from the four corners is 1 inch, the resulting box has volume 36 cubic inch.

Example 1: Modeling the Volume of a Box

After experimenting, Tuan discovers that 2 lb of the popcorn kernels occupies a volume of 25 cubic inches.

What is the dimension of a box to hold 2 *lb* of the popcorn kernels?



Example 3 p. 211: Maximizing Revenue from Ticket Sales

A hockey team plays in an arena with a seating capacity of 15,000 spectators. With the ticket price set at \$14, average attendance at recent games has been 9500. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000.

a) Find a function that models the revenue in terms of ticket price.
b) What ticket price is so high that no one attends, and hence no revenue is generated?
c) Find the price that maximizes revenue from ticket sales.

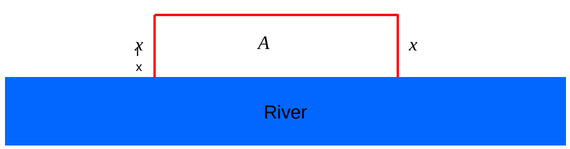
revenue = ticket price × attendance
ticket price = x
attendance = $1000(14 – x) + 9500$
revenue = $x(1000(14 – x) + 9500)$

Problem 23 on Page 215

Consider the following problem: A farmer has 2400 ft of fencing and wants to fence off a rectangle field that borders a straight river. He does not need a fence along the river. (See the figure.) What are the dimensions of the field of the largest area that he can fence?

- Experiment with the problem by drawing several diagrams illustrating the situation. Calculate the area of each configuration, and use your result to estimate the largest possible field.

Area = length × width
width = x
length = $2400 – 2x$
Area = $x(2400 – 2x)$

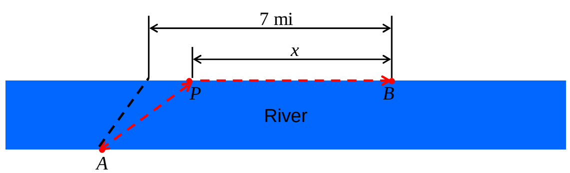


Problem 34 on Page 217

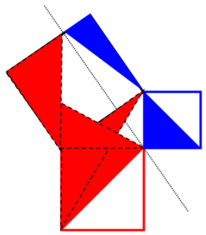
A man stands at a point *A* on the bank of a straight river, 2 mi wide. To reach point *B*, 7 mi downstream on the opposite bank, he first rows his boat to point *P* on the opposite bank and then walks the remaining distance *x* to *B*, as shown in the figure. He can row at a speed of 2 mi/h and walk at a speed of 5 mi/h.

a) Find a function that models the time needed for the trip.
b) Where should he land so that he reaches *B* as soon as possible?

total time = rowing time + walking time = *AP* × rowing speed + *PB* × walking speed
$AP = \sqrt{(7 – x)^2 + 2^2} = \sqrt{(7 – x)^2 + 4}$
rowing time = $2\sqrt{((7 – x)^2 + 4)}$
walking time = $5x$



Pythagorean Proof



Practice Problems Page 214

1-14, 16-18, 23b, 24a, 28a, 29a, 34a.

Combining Functions

Algebra of Functions

Let *f* and *g* be functions with domains *A* and *B*. Then the function $f+g$, $f-g$, fg, and f/g are defined as follows

$(f+g)(x) = f(x)+g(x)$ Domain $A \cap B$

$(f-g)(x) = f(x)-g(x)$ Domain $A \cap B$

$(fg)(x) = f(x)g(x)$ Domain $A \cap B$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ Domain $\{x \in A \cap B \mid g(x) \neq 0\}$

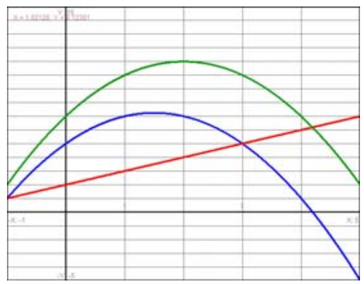
Example

$f(x) = -x^2 + 3x + 5$

$g(x) = x + 2$

$(f + g)(x) = ?$

$(f + g)(x) = -x^2 + 4x + 7$



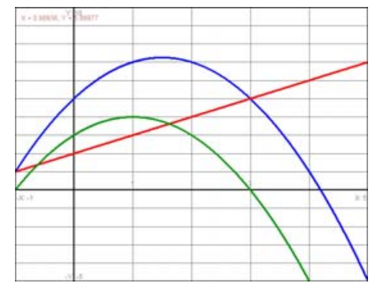
Example

$f(x) = -x^2 + 3x + 5$

$g(x) = x + 2$

$(f - g)(x) = ?$

$(f - g)(x) = -x^2 + 2x + 3$



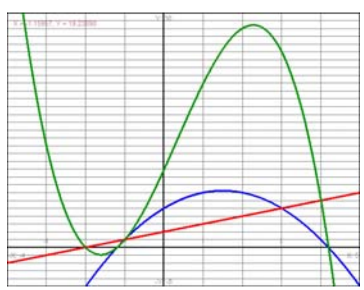
Example

$f(x) = -x^2 + 3x + 5$

$g(x) = x + 2$

$(fg)(x) = ?$

$(fg)(x) = -x^3 + x^2 + 11x + 10$



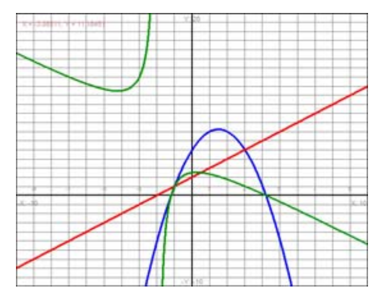
Example

$f(x) = -x^2 + 3x + 5$

$g(x) = x + 2$

$\left(\frac{f}{g}\right)(x) = ?$

$\left(\frac{f}{g}\right)(x) = \frac{-x^2 + 3x + 5}{x + 2}$



Problem on Page 225

Find the domain of the function

7. $f(x) = \sqrt{x} + \sqrt{1-x}$

8. $f(x) = \sqrt{x+1} - \dfrac{1}{x}$

Composition of Functions

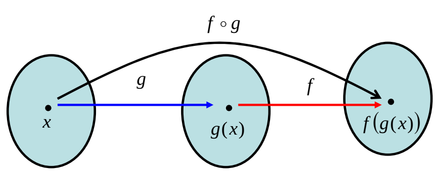
Given two functions *f* and *g*, the *composite function* $f \circ g$ or composition of *f* and *g* is defined by

$$(f \circ g)(x) = f(g(x))$$

Composition of Functions

Given two functions *f* and *g*, the *composite function* $f \circ g$ or *composition* of *f* and *g* is defined by

$$(f \circ g)(x) = f(g(x))$$

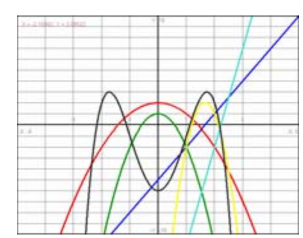


Example on Page 225

Use $f(x) = 3x - 5$ and $g(x) = 2 - x^2$ to evaluate the expression.

18. a. $f(f(4))$ b. $g(g(3))$

21. a. $(f \circ g)(x)$ b. $(g \circ f)(x)$



Example on Page 227

You have a \$50 coupon from the manufacturer good for the purchase of a cell phone. The store where you are purchasing your cell phone is offering a 20% discount on all cell phones. Let x represent the regular price of the cell phone.

a) Suppose only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the regular price x.

b) Suppose only the \$50 coupon applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x.

c) If you can use the coupon and the discount, then the purchase price is either $(f \circ g)(x)$ or $(g \circ f)(x)$ depending on the order in which they are applied to the price. Find both $(f \circ g)(x)$ and $(g \circ f)(x)$ Which composition gives the lower price?

Practice Problems on Page 225

1,2,6,11,17,19,22,23-28,29,33,40,45-50,55,56,60.

One-to-One Functions and Their Inverse

Definition of a One-to-One Function

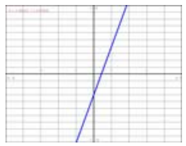
A function within domain A is called a *one-to-one function* if no two elements of A have the same output, that is

$$f(x_1) \neq f(x_2) \quad \text{where} \quad x_1 \neq x_2$$

Example on Page 237

Determine whether the function is one-to-one.

7. $f(x) = 7x - 3$



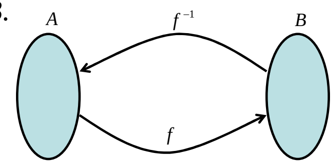
8. $f(x) = x^2 - 2x + 5$

Definition of the Inverse of a Function

Let *f* be a one-to-one function with domain *A* and range *B*. Then its *inverse function* f^{-1} has domain *B* and range *A* and its defined by

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

for any *y* in *B*.



Problems on Page 237

Find the inverse function of *f*.

33. $f(x) = 4x + 7$

37. $f(x) = \dfrac{1}{x+2}$

44. $f(x) = \sqrt{2x - 1}$

Property of Inverse Functions

Let *f* be a one-to-one function with domain *A* and range *B*. The inverse function f^{-1} satisfies the following cancellation properties.

$$f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A$$

$$f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B$$

Conversely, any function f^{-1} satisfying these equations is the inverse of *f*.

Problems on Page 237

Use the property of inverse function to show that f and g are inverse of each other.

21. $f(x) = x + 3, \quad g(x) = x - 3$

27. $f(x) = x^2 - 4, \quad x \geq 0;$

$g(x) = \sqrt{x+4}, \quad x \geq -4;$

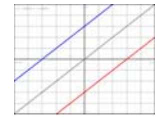
Problems on Page 237

Use the property of inverse function to show that f and g are inverse of each other.

21. $f(x) = x + 3, \quad g(x) = x - 3$

27. $f(x) = x^2 - 4, \quad x \geq 0;$

$g(x) = \sqrt{x+4}, \quad x \geq -4;$



Practice Problems on Page 236

1-6,23,24,31,34,38,39,41,42,67,70.

Fitting Lines to Data

Fitting Lines to Data

A *mathematical model* is usually a function that describes the behavior of a certain process.

- One way to obtain a mathematical model is to use mathematical fitting data technique to a set of available data.
- We will focus on fitting lines to data.

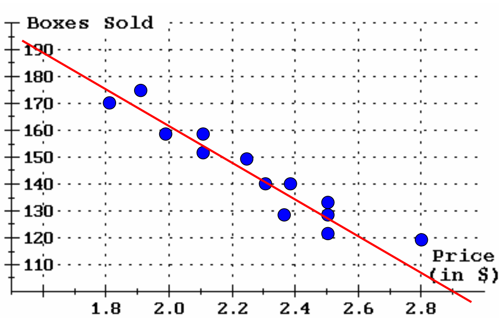
Cereal Example

A cereal manufacturer is developing a new cereal, Ketchup-Flavored Corn Flakes. The manufacturer test-marketed the cereal at various prices at several stores. The data obtained from the test are given in the following table and will be used to predict the weekly sales of the cereal based upon the price per box. For example, at one store at a price of $2.30 per box, sales of the cereal were 140 boxes per week, whereas at another store at a price of $2.10 per box, the sales were 160 boxes per week.

Price per box (in $)	2.30	2.10	1.80	1.90	2.50	2.80	1.99	1.90	2.25	2.39	2.37	2.10	2.50	2.50
Number of boxes sold	140	160	170	175	130	120	160	170	150	140	130	152	134	122

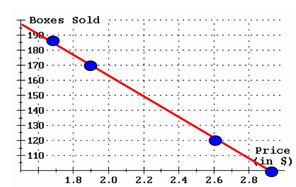
Price per box (in $)	2.30	2.10	1.80	1.90	2.50	2.80	1.99	1.90	2.25	2.39	2.37	2.10	2.50	2.50
Number of boxes sold	140	160	170	175	130	120	160	170	150	140	130	152	134	122

Plot a point in the figure for each data pair given in the table .



Cereal Example

- **Use the line drawn in the graph to predict weekly sales of the cereal at various prices.**



Price per box	**$2.60**	**$1.85**	**$1.70**	**$2.90**
Predicted Weekly sales	**120**	**170**	**187**	**100**

Cereal Example

- Choose symbolic labels for the straight line function on the graph, as well as the input quantity and the output quantity. We will call the function h, the price per box p, and the number of boxes sold n. Then we write

$$n = h(p)$$

- What is the price per box of the cereal if 131 of boxes sold?

$$h(2.50) = 131$$

Interpolation and Extrapolation

- *Interpolation* is an estimation of an output value corresponding to the input value inside the values from the given set of data.

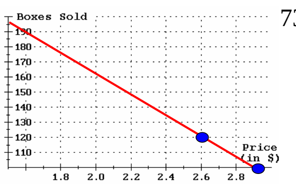
$$n = h(1.85) = 170$$
$$n = h(2.60) = 120$$

- *Extrapolation* is an estimation of an output value corresponding to the input value outside the values from the given set of data.

$$n = h(1.70) = 187$$
$$n = h(2.90) = 100$$

Terminology

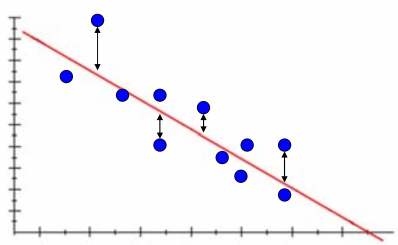
- A *scatter plot* of data is set of points representing data on a graph. Example.
- Fitting the best line to a set of data is called *linear regression*.
  - Example: The regression line is $283.55 - 61.04x$
  - Correlatio[n] 7391304348



How Regression Works

How does a calculator or computer decide which line fits the data best?

Least-squares or regression line: The line that minimizes the sum of the squares of the vertical distances between the data points and the line.

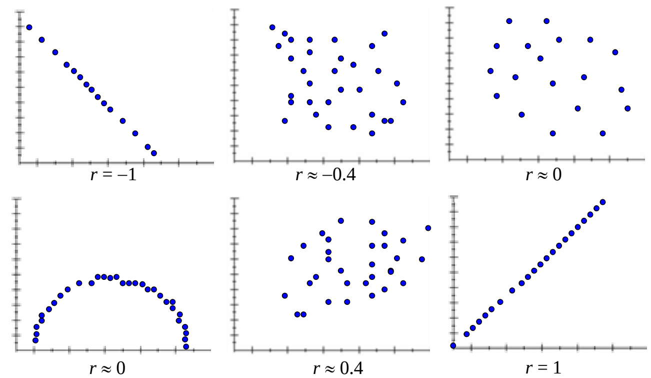


How Good is the Fit?

A *correlation coefficient*, $r \in [-1, 1]$ is used to measure how well a particular regression line fits the data.

- If $r = 1$, the data lie exactly on a line of positive slope.
- If $r = -1$, the data lie exactly on a line of negative slope.
- If r is close to 0, the data may be completely scattered, or they may be a non-linear relationship between the variable.

Examples



Practice Problems on Page 250

1,2,4