Exercises Section 2.3 [Page 65]

29. As of April 2006, roughly 50 million .com web domain names were registered (e.g., yahoo.com).
a. How many domain names consisting of just two letters in sequence can be formed? How many domain names of length two are there if digits as well as letters are permitted as characters? [*Note*: A character length of three or more is now mandated.]

b. How many domain names are there consisting of three letters in sequence? How many of this length are there if either letters or digits are permitted? [*Note:* All are currently taken.]

c. Answer the questions posed in (b) for four-character sequences.

d. As of April 2006, $97,786$ of the four-character sequences using either letters or digits had not yet been claimed. If a four-character name is randomly selected, what is the probability that it is already owned?

30. A friend of mine is giving a dinner party. His current wine supply includes
8 bottles of zinfandel,
10 of merlot, and
12 of cabernet (he only drinks red wine),
all from different wineries.

a. If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?

b. If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?

c. If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?

d. If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?

e. If 6 bottles are randomly selected, what is the probability that all of them are the same variety?

31. a. Beethoven wrote 9 symphonies and Mozart wrote 27 piano concertos. If a university radio station announcer wishes to play first a Beethoven symphony and then a Mozart concert, in how many ways can this be done?

b. The station manager decides that on each successive night (7 days per week), a Beethoven symphony will be played, followed by a Mozart piano concerto, followed by a Schubert string quartet (of which there are 15). For roughly how many years could this policy be continued before exactly the same program would have to be repeated?

32. A stereo store is offering a special price on a complete set of components (receiver, compact disc player, speakers, cassette deck). A purchaser is offered a choice of manufacturer for each component:

Receiver: Kenwood, Onkyo, Pioneer, Sony, Sherwood
Compact disc player: Onkyo, Pioneer, Sony, Technics
Speakers: Boston, Infinity, Polk
Cassette deck: Onkyo, Sony, Teac, Technics

A switchboard display in the store allows a customer to hook together any selection of components (consisting of one of each type). Use the product rules to answer the following questions:
a. In how many ways can one component of each type be selected?

b. In how many ways can components be selected if both the receiver and the compact disc player are to be Sony?

c. In how many ways can components be selected if none is to be Sony?

d. In how many ways can a selection be made if at least one Sony component is to be included?

e. If someone flips switches on the selection in a completely random fashion, what is the probability that the system selected contains at least one Sony component? Exactly one Sony component?

35. A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the grave-yard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

a. How many selections result in all 6 workers coming from the day shift? What is the probability that all 6 selected workers will be from the day shift?

b. What is the probability that all 6 selected workers will be from the same shift?

c. What is the probability that at least two different shifts will be represented among the selected workers?

d. What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

38. A box in a certain supply room contains
four 40-W lightbulbs,
five 60-W bulbs, and
six 75-W bulbs.
Suppose three bulbs are randomly selected.

a. What is the probability that exactly two of the selected bulbs are rated 75 W?

b. What is the probability that all three of the selected bulbs have the same rating?

c. What is the probability that one bulb of each type is selected?

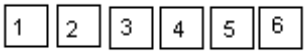
d. Suppose now that bulbs are to be selected one-by-one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

42. Three married couples have purchased theater tickets and are seated in a row consisting of just six seats. If they take their seats in a completely random fashion (random order), what is the probability that Jim and Paula (husband and wife) sit in the two seats on the far left? What is the probability that Jim and Paula end up sitting next to one another? What is the probability that at least one of the wives ends up sitting next to her husband?

43. In five-card poker, a straight consists of five cards with adjacent denominations (e.g., 9 of clubs, 10 of hearts, jack of hearts, queen of spades, and king of clubs).
Assuming that aces can be high or low, if you are dealt a five-card had, what is the probability that it will be a straight with high card 10? What is the probability that it will be a straight? What is the probability that it will be a straight flush (all cards in the same suit)?

Hint to exercise 42 in section 2.3.

42. Seats:



a. P(J&P in 1&2) $= \dfrac{2\times1\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1} = \dfrac{1}{15}$

b. P(J&P next to each other) = P(J&P in 1&2) + … + P(J&P in 5&6)

$= 5\times\dfrac{1}{15}$

c. Use the complement.
P(at least one H next to his W) = 1 – P(no H next to his W)

Next, count the # of ways of no H next to his W in two parts as follows.

I. # of orderings with a H-W pair in seats #1 and 3 and no H next to his W
$= 6^{*} \times 4 \times 1^{*} \times 2^{\#} \times 1 \times 1 = 48$
*= pair, # =can't put the mate of seat #2 here or else a H-W pair would be in #5 and 6.

II. # of orderings without a H-W pair in seats #1 and 3, and no H next to his W
$= 6 \times 4 \times 2^{\#} \times 2 \times 2 \times 1 = 192$
#= can't be mate of person in seat #1 or #2.

So, by combining the results from I and II,
# of seating arrangements with no H next to W = 48 + 192 = 240.

Therefore,
P(no H next to his W) = $= \dfrac{240}{6\times5\times4\times3\times2\times1} = \dfrac{1}{3}$,
and
P(at least one H next to his W) = = 1 – P(no H next to his W) = $1 - \dfrac{1}{3} = \dfrac{2}{3}$